Dynamic Information Aggregation: Learning from the Past[⋆]

Zhen Huo[a,*], Marcelo Pedroni[b]

[a] *Yale University, USA*
[b] *University of Amsterdam, Netherlands*

Abstract

With dispersed information, how much can agents learn from past endogenous aggregates such as prices or output? In a rational-expectations equilibrium, if general equilibrium effects are strong enough, aggregates no longer perfectly reveal underlying fundamentals. In this confounding regime, the effects of informational frictions are persistent over time, and the aggregate outcome displays an initial under-reaction followed by a delayed over-reaction relative to its perfect-information counterpart. In a standard New Keynesian model, we show that endogenous information aggregation helps bring the model predictions on aggregate forecasts closer to the data.

Keywords: Information Aggregation, Dispersed Information, Monetary Policy

1. Introduction

In this paper, we revisit a number of classical questions in the information literature: How much can agents learn from endogenous aggregate outcomes about underlying fundamentals? How does the informativeness of aggregate statistics depend on the degree of strategic complementarity between agents? Can monetary policy generate persistent real effects when the history of inflation is perfectly observed? We explore these questions in economies with persistent information and strategic interactions between agents.

We start with a canonical beauty-contest model in which agents care about some persistent economic fundamental and also an endogenous aggregate outcome. We assume agents receive

⋆We are grateful to two anonymous referees and the editor, Yuriy Gorodnichenko, for helpful feedback, and to Yan Xiong for discussing our paper in the 2019 PHBS Workshop in Macroeconomics and Finance. The paper benefits from comments from Marios Angeletos, Ryan Chahrour, Simas Kucinskas, Keith Kuester, Jeniffer La'O, Paul Levine, Giacomo Rondina, Christian Stoltenberg, Naoki Takayama, Todd Walker, and Pengfei Wang. We would also like to thank the seminar participants at PHBS, Università Cattolica del Sacro Cuore, University of Bonn, TI Macro Conference in Rotterdam. We thank Flint O'Neil for providing excellent research assistance. Lastly, we thank Zhao Han, Fei Tan, and Jieran Wu for generously sharing their computational results with us.

*Corresponding author
Email address: zhen.huo@yale.edu (Zhen Huo)

two types of signals: private noisy signals about current fundamentals and the perfect observation of the history of past aggregate outcomes. The lack of observation of the current aggregate outcome captures the idea that firms and households often make their production and consumption decisions before contemporaneous aggregate statistics are available. We show that in a rational expectations equilibrium, two possible regimes can arise endogenously: a revealing regime and a confounding regime.[1] In the revealing regime, the history of outcomes perfectly aggregates information, and agents can infer the underlying fundamental without error. This is in line with the conventional wisdom that, even with dispersed information, prices or output help eliminate uncertainty about the aggregate fundamental. However, there also exists a confounding regime in which agents make persistent forecast errors, and consensus about the fundamental is never achieved.

Whether the confounding regime arises depends on the degree of strategic complementarity between agents, which can be interpreted as the strength of general equilibrium (GE) feedback effects, and on the precision of private signals about the fundamental. Endogenous outcomes aggregate information only if agents respond to their private signals which are direct sources of information. The more agents rely on past aggregate outcomes in their inference, the less information contained in private signals can be encoded into individuals' actions, resulting in potential information dilution. The extent to which agents rely on the history of aggregate outcomes is decreasing in the precision of private signals and increasing in the need to coordinate with other agents.[2] In an environment with persistent information, a sufficient amount of information dilution can make the process of the aggregate outcome non-invertible. This insight extends to various environments with multiple signals, forward-looking beauty-contest games, and multivariate systems.

The dynamics of aggregate outcomes also differ in a significant way under these two regimes. In the revealing regime, in response to an innovation to the fundamental, the aggregate outcome underreacts on impact, since information about the current fundamental is dispersed. Afterwards, the aggregate outcome exactly follows the fundamental since, by observing past aggregate outcomes, agents reach common knowledge about the innovation. In contrast, in the confounding regime, past outcomes are not sufficient for agents to infer the underlying innovations, and

[1] The notion of confounding dynamics that results from a non-invertible process was first introduced in an early version of Rondina and Walker (2021).

[2] See a more detailed discussion on this point in Morris and Shin (2002), Angeletos and Pavan (2007), and Huo and Pedroni (2020).

agents' forecast errors are persistent over time. Still, the observation of past aggregate outcomes does not allow for the systematic under- or overestimation of the fundamental. As a result, the aggregate outcome fluctuates around the fundamental, alternating between over-reaction and under-reaction. Furthermore, we prove that the process for the aggregate outcome is complex in the sense that it does not permit a finite-state representation.

An implication of the two informational regimes is that there exists a "kink" on how the informativeness of aggregate outcomes is related to the structural parameters. Within the revealing regime, a local variation of parameters only affects the impact response of the aggregate outcome but does not change its response afterwards nor its invertibility. In response to a large enough shift in the parameters, the economy enters the confounding regime, in which case either a larger strategic complementarity or a lower signal precision decrease the informativeness of the aggregate outcome.

Our baseline specification focuses on square information systems with the number of signals equal to the number of underlying shocks. It is well-established that, when there are fewer signals than shocks, economic agents cannot determine the underlying shocks. With square information structures, the ability of the economy to properly aggregate information then depends on the deep parameters of the model, and our results contribute to formalize this relationship. Our results can also be viewed as a benchmark to understand the behavior of models with non-square information structures. By continuity, the predictions in our baseline model remain valid when low precision signals or low variance shocks are added to a square information structure. One can also smoothly move from our endogenous information benchmark to a fully exogenous information environment by adding noise to endogenous signals.

We apply our results to New Keynesian models with incomplete information. We show that with dispersed information, the confounding regime is more likely to arise with a lower nominal rigidity in the New Keynesian Phillips curve and a higher marginal propensity to consume (MPC) in the dynamic IS curve. With more flexible prices, firms that can adjust their prices need to pay more attention to others' prices; with a higher MPC, consumers are more sensitive to changes in the aggregate demand. Both of these map into stronger GE feedback effects which manifest as stronger degrees of strategic complementarity, and hence lead to non-invertible aggregate outcomes.

When the central bank employs a Taylor-type monetary policy, a more aggressive response of nominal interest rate to inflation reduces the positive feedback effects between the supply and demand block, resulting in a lower degree of strategic complementarity. With incomplete information, a more aggressive policy not only modifies the magnitude of the economy's response to shocks, but it can also alter the entire dynamic pattern by shifting the economy from the confounding to the revealing regime. It follows that a dovish monetary policy rule tends to induce hump-shaped and oscillatory responses of inflation and output.

Finally, we bring the New Keynesian model to the data and explore whether endogenous information can help account for the observed patterns in aggregate forecasts. We calibrate informational frictions to match the impulse responses of the average forecast about unemployment from the Survey of Professional Forecasters to the main business cycle shock identified in Angeletos, Collard, and Dellas (2020). In the data, the response of aggregate forecasts is initially dampened relative to the outcome, but overshoots the outcome later on. It follows that the forecast errors display a sign-switching pattern (Angeletos, Huo, and Sastry, 2020). Under our calibration, the endogenous outcome follows a non-invertible process, which limits the amount of information agents can learn from past outcomes. Consequently, the aggregate forecasts under-react to shocks, which is also consistent with the evidence provided in Coibion and Gorodnichenko (2015). In addition, learning from past outcomes yields the aforementioned oscillatory dynamics, which naturally gives rise to the sign switching pattern of the forecast error observed in the data. This also helps the model meet the evidence, documented by Kohlhas and Walther (2021), that points to a negative correlation between the forecast error and the outcome: when the forecast error turns from positive to negative, the correlation between the forecast error and the aggregate outcome is dulled. In contrast, in the model where past shocks are public information, the dampening of the aggregate forecasts is rather weak compared with data. In the model where agents only receive exogenous noisy information, the forecast error is always positive and never switch signs.

Related Literature

The study of how well endogenous variables like prices summarize relevant dispersed information dates back to Hayek (1945). What makes information aggregation nontrivial in our model is the fact that fundamentals and signals are correlated over time. Our work therefore complements the line of research that focuses on endogenous information aggregation that relies on the history of signals, including, for instance, Townsend (1983), Sargent (1991), Kasa (2000), Nimark (2014), and Bacchetta and Wincoop (2006). This paper is closely related to Rondina (2008), Graham and Wright (2010), Baxter, Graham, and Wright (2011), Kasa, Walker, and Whiteman (2014), Han, Ma, and Mao (2022), and Rondina and Walker (2021), who also explore non-invertible equilibrium dynamics in various settings. Relative to these papers, we provide a condition for equilibrium invertibility in a general class of models and explore how it relates to the degree of complementarity. In addition, whereas the equilibrium dynamics in the aforementioned papers follow tractable finite-state processes,[3] in our paper, the dynamics is more involved and we prove it does not permit a finite-state representation.[4]

This paper is also related to the large literature on monetary non-neutrality due to informational frictions. Lucas (1972) showed this is consistent with rational expectations. In that setup, real effects of monetary shocks were predicated on variations in aggregate nominal expenditure not being forecastable. It follows that, if data on past aggregates were available these effects could, at most, be transitory. Woodford (2003), following Sims (2003), argues that the main informational bottleneck is not its plain availability but the "limited capacity of private decision-makers to pay attention" to it. Then, assuming aggregates are never fully observed, he shows that monetary shocks can have persistent effects. One could also generate persistent effects by including additional exogenous aggregate shocks in the model or imposing rational inattention, as in Nimark (2008), Adam (2007), Maćkowiak and Wiederholt (2009), and Melosi (2017), or endogenous sentiment shocks, as in Acharya, Benhabib, and Huo (2021). In this paper, we show that even if past aggregates are perfectly observed, monetary policy can have long-lasting effects which arise endogenously only if GE effects are strong enough.[5]

A common assumption in the literature is that information is static or the underlying states are perfectly revealed after one period, which allows one to focus on the within-period inference problem. Grossman (1976) and Hellwig (1980) studied to what extent prices can summarize multidimensional information in a static setting. Messner and Vives (2005), Angeletos and Pavan (2009), Amador and Weill (2010, 2012) and Vives (2014, 2017) study learning externalities associated with the observation of endogenous aggregates. A particular type of learning externality plays an important role in deviations from perfect information aggregation in our model: strategic complementarity leads agents to respond less to their private information, which has the negative side effect of reducing the informativeness of aggregates in general and prices in

[3]Baxter, Graham, and Wright (2011) solves the model numerically and the impulse response functions in their paper do not display the oscillation pattern as in our paper.

[4]This is the case even for square information structures—when the number of signals is equal to the number of shocks—which usually lead to more tractable equilibria.

[5]An exception is Hellwig and Venkateswaran (2015). They identify special cases in which dispersed information is irrelevant for allocation.

particular. In persistent information settings as the one considered in our paper, this crowding out can lead to non-invertibility and oscillatory dynamics. Angeletos, Iovino, and Jennifer (2020) explore optimal policy under this type of learning externality. They find that agents are better off when incentivized to respond more aggressively to their belief variations, which helps to achieve better information aggregation. Gaballo (2018) and Chahrour and Gaballo (2021) show that learning from prices can play either a propagating or a mitigating role. Their mechanism relies on agents receiving different types of signals, while our framework focuses on the classical case with a common information structure.

2. A Monetary Model

In this section, we introduce a simple model of monopolistic competition to illustrate the basic idea of information aggregation through endogenous actions. The model is largely borrowed from Woodford (2003)[6] and it is designed not to exhaust the implications of our main results but to be the simplest setup in which we can highlight the role of strategic complementarity in determining the information content of endogenous information. It allows us to examine the question of how efficiently prices and other macro variables aggregate the information that firms need to make their decisions.

Suppose the producer of good i chooses the current and future prices of their good, $\{P_{it+k}\}$, to maximize the expected present value of future (real) profits

$$\mathbb{E}_{i,t}\left[\sum_{k=0}^{\infty}\beta^k m(Y_{t+k})\left[\frac{P_{it+k}}{P_{t+k}}Y_{i,t+k} - C(Y_{i,t+k}, Y_{t+k})\right]\right]$$

subject, in every period $t+k$, to the demand function

$$Y_{i,t+k} = Y_{t+k}\left(\frac{P_{i,t+k}}{P_{t+k}}\right)^{-\theta},$$

for some $\theta > 1$. The producer takes the Dixit-Stiglitz index of real aggregate demand, Y_t, and the corresponding price index, P_t, as given. The real cost of producing is given by $C(Y_{it}, Y_t)$ and depends not only on the amount produced by the firm, Y_{it}, but also on the aggregate output, Y_t, to account for its effect on factor prices. The firm weights different states by the stochastic discount factor, $m(Y_t)$, so that the expected discounted profits can be interpreted as a financial market valuation of the firm.

[6]The main difference relative to Woodford (2003) is that we consider a more general information structure.

To focus on informational frictions we abstract from price frictions here. The firms can choose their prices every period independently of the past. Thus, in period t, firm i chooses P_{it} to maximize

$$\mathbb{E}_{it}\left[Y_t\left(\frac{P_{it}}{P_t}\right)^{1-\theta} - C\left(Y_t\left(\frac{P_{it}}{P_t}\right)^{-\theta}, Y_t\right)\right],$$

where $\mathbb{E}_{it}$ denotes the expectation conditional on the information set of firm i in period t. The optimal pricing decision of firm i implies a first order condition which can be log-linearly approximated by

$$p_{it} = \mathbb{E}_{it}[(1-\alpha)y_t + p_t],$$

where lower-case variables denote log-deviations from the full-information steady-state versions of the corresponding upper-case variables, which satisfy $P_{it} = P_t$ and $Y_t = Y^*$. The parameter α can be written as a function of deep parameters.[7]

Changes in nominal aggregate demand, $Q_t = P_t Y_t / Y^*$, can be decomposed into changes in the price level and in the real output,

$$q_t = p_t + y_t.$$

We assume that the nominal aggregate demand is determined exogenously by the central bank, following an AR(1) process[8]

$$q_t = \rho q_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0,1).$$

Therefore, though the innovations to the nominal aggregate demand, η_t, can have a broader interpretation, from now on, we refer to them as monetary shocks.

From an individual firm's perspective, their pricing decision can be expressed as the best-response function of a standard static beauty-contest game, that is

$$p_{it} = (1-\alpha)\mathbb{E}_{it}[q_t] + \alpha\mathbb{E}_{it}[p_t], \quad \text{with} \quad p_t = \int p_{it}\,\mathrm{d}i,$$

and where $\alpha \in (-1,1)$ controls the degree of strategic complementarity between firms, or the strength of GE feedback effects—we use both terms interchangeably.

With incomplete information, the aggregate price depends not only on the firms' first-order expectations about the nominal aggregate demand but also on higher-order expectations as emphasized in Morris and Shin (2002) and Woodford (2003). We formalize this in the following lemma.

[7]More specifically, Y^* solves $(1-\theta) + \theta C_y(Y^*, Y^*) = 0$, and $\alpha \equiv 1 - \frac{C_{yy}(Y^*,Y^*)Y^* + C_{yY}(Y^*,Y^*)Y^*}{C_y(Y^*,Y^*) + \theta C_{yy}(Y^*,Y^*)}$.

[8]We relax this assumption in Section C.

Lemma 2.1. *The aggregate price is given by*

$$p_t = (1-\alpha)\sum_{k=0}^{\infty}\alpha^k \overline{\mathbb{E}}_t^{k+1}[q_t], \tag{1}$$

where the higher-order expectations are defined recursively as follows

$$\overline{\mathbb{E}}_t^0[q_t] \equiv q_t, \quad \textit{and} \quad \overline{\mathbb{E}}_t^{k+1}[q_t] \equiv \int \mathbb{E}_{it}\left[\overline{\mathbb{E}}_t^k[q_t]\right] \mathrm{d}i.$$

Proof. See Appendix A.1.1. □

The real effects of a monetary shock on output are captured by the gap between p_t and q_t. Condition (1) implies that this gap depends on the dynamics of all higher-order expectations, which in turn depend on the information structure faced by firms. To proceed, we first consider a benchmark case with dispersed but *exogenous* information, in which the information content is independent of equilibrium objects. In the next section, we turn to the *endogenous* information case, in which the information content is a function of equilibrium objects.

Exogenous-Information Benchmark. Suppose that every period, t, firm i receives a new private signal, x_{it}, about the nominal aggregate demand[9]

$$x_{it} = q_t + u_{it}, \quad u_{it} \sim \mathcal{N}(0, \tau^{-1}). \tag{2}$$

With $\tau \to \infty$, we return to the frictionless case where firms observe nominal aggregate demand, q_t, perfectly. Not only is there no first-order uncertainty about the fundamental, but also all higher-order expectations become common knowledge and collapse to the first-order expectation, that is, $\overline{\mathbb{E}}_t^k[q_t] = q_t$ for all $k \geq 0$. It follows that

$$p_t^* = q_t, \qquad y_t^* = 0.$$

Thus, prices vary with the nominal aggregate demand in a one-to-one fashion, leaving aggregate output unchanged.

With $\tau < \infty$, information is incomplete, and the equilibrium outcome is shaped by the complicated dynamics of all the higher-order expectations in condition (1). We can, however, bypass this complexity by applying the single-agent result in Huo and Pedroni (2020), which directly yields a simple characterization of the price dynamics that follows an AR(2) process:[10]

$$p_t = \left(1 - \frac{\vartheta}{\rho}\right)\frac{1}{1-\vartheta L} q_t, \tag{3}$$

[9] This information structure is similar to the ones used in Woodford (2003) and Angeletos and La'O (2010).

[10] This price dynamics is similar to the one obtained in Rondina (2008) where individuals' best responses depend on idiosyncratic fundamentals instead of a common one.

where $\vartheta \in (0, \rho)$ is given by

$$\vartheta = \frac{1}{2}\left(\rho + \frac{1+(1-\alpha)\tau}{\rho} - \sqrt{\left(\rho + \frac{1+(1-\alpha)\tau}{\rho}\right)^2 - 4}\right).$$

The aggregate price moves less than the nominal aggregate demand, captured by the term $1 - \vartheta/\rho$ in equation (3), which implies that real output co-moves with its nominal counterpart. In addition, it takes time for firms to learn the monetary shock; and the deviation of prices from the nominal aggregate demand is persistent, captured by the term $1/(1 - \vartheta L)$. This leads to long-lasting effects of monetary shocks on real output.[11]

Perfect-Revealing Benchmark. Another useful benchmark is to allow agents to observe past shocks perfectly in addition to the exogenous noisy signals, as in Lucas (1972). In this case, uncertainty about the underlying state is only contemporary, being resolved in the next period. The price dynamics follows

$$p_t = \frac{(1-\alpha)\tau}{1+(1-\alpha)\tau}\eta_t + q_{t-1}. \tag{4}$$

Except for the current innovation, this price process tracks the nominal aggregate demand exactly. The dampened response on impact reflects imperfect knowledge about the current innovation and the coordination motive. It follows that monetary shocks have only transitory effects on real output.

3. Endogenous Information Aggregation

In this section, we allow the signals in firms' information set to include endogenous aggregate variables, so that the informativeness of the information is determined in equilibrium. We show that this change can lead to significantly different price and output dynamics, and affects how much can be learned from observing aggregate variables.

Information Structure. We assume that in any period t, signals in firms' information set include

1. private exogenous signals, $\{x_{i,t}, x_{i,t-1}, \ldots\}$ as in (2);
2. past prices, $\{p_{t-1}, p_{t-2}, \ldots\}$.

[11] In Angeletos and Huo (2021), aggregate outcomes follow similar dynamics in a forward-looking game, where the determination of ϑ is more complicated.

The first set of signals makes information dispersed as mentioned before, while the second set of signals allow agents to better coordinate and learn from each others' actions.

There are two main reasons why we assume agents do not observe the contemporaneous aggregate outcomes but rather their delayed values: First, from a practical point of view, firms and households typically need to plan their production or purchasing choices in advance of the realizations of fundamentals like demand shocks or income. There are lags in the publication of aggregate statistics as well, and nowcasts are necessary for most aggregate variables probably with the exception of frequently traded asset prices. Second, from a theoretical point of view, the simultaneity of making an individual decision while observing the aggregate outcome is easier to be justified when the aggregate states are perfectly observable and agents can form rational expectations about the aggregate variables without frictions. In our economy with dispersed information, observing past outcomes is a more natural timing of decisions as aggregate states are observed with noise.

To measure the amount of information aggregated by price, we define price informativeness as

$$\chi \equiv 1 - \frac{\mathrm{Var}\left[q_t - \mathbb{E}\left[q_t|p^t\right]\right]}{\mathrm{Var}[q_t]},$$

where p^t denotes the history of price observations $\{p_t, p_{t-1}, \dots\}$. Notice that $\chi \in [0,1]$. When prices perfectly reveal the nominal aggregate demand, the mean squared prediction error is zero and price informativeness reaches its maximum level, $\chi = 1$. When prices are not informative at all, the forecast $\mathbb{E}\left[q_t|p^t\right] = 0$ and $\chi = 0$.

3.1. Invertibility

We start with a thought experiment in which only a single firm can observe past prices. As this firm is infinitesimal, the aggregate price process is the same as in the exogenous information case. How much can this firm learn from the history of prices? To answer this question, we rearrange condition (3) to obtain

$$q_{t-1} = \left(1 - \frac{\vartheta}{\rho}\right)^{-1} (p_{t-1} - \vartheta p_{t-2}). \tag{5}$$

That is, by observing the history of prices, the uncertainty about *past* shocks is completely eliminated. Price informativeness is, then, at its maximal level, $\chi = 1$.

This is the basic logic behind the conventional wisdom that prices can effectively aggregate information. In the context of monetary neutrality, it would imply that when firms observe past prices, the monetary shock can only have transitory effects. However, this logic does not necessarily extend to the case in which all firms can observe past prices, since the different information structure leads to different equilibrium price dynamics. Importantly, the mapping from prices to underlying shocks obtained in condition (5) may not be possible. What determines if a price process is able to perfectly aggregate information is its invertibility, as we formalize in the next lemma.

Lemma 3.1. *A price process $p_t = g(L)\eta_t$ is invertible if $g(L)$ does not contain any inside root.*[12] *When invertible, prices perfectly aggregate information about the underlying shock, i.e.,*

$$\mathbb{E}[q_{t-k}|p^t] = \frac{g(L)^{-1}p_{t-k}}{1-\rho L} = q_{t-k}, \quad \text{for } k \geq 0. \tag{6}$$

Proof. See Appendix A.1.2. □

Corollary 3.1. *With exogenous information, for all admissible parameters (α, τ, ρ), the price process is always invertible and the history of prices contains the same information.*

The aforementioned exogenous information case is an example of an invertible process. Perhaps somewhat surprisingly, even with very large noise (small τ), the history of prices still reveals all the information about the monetary shock, even though the magnitude of the price response can be arbitrarily small.

3.2. Two Regimes of Information Aggregation

When information is endogenous and dispersed, the extent to which prices efficiently aggregate information hinges both on the severity of informational frictions and, crucially, on the degree of strategic complementarity.

Proposition 3.1. *The equilibrium price perfectly aggregates information, that is, $\mathbb{E}[q_t|p^t] = q_t$, if and only if the triple (α, τ, ρ) is in the invertible region that satisfies*

$$\alpha < 1 - \frac{\rho}{\tau}. \tag{7}$$

Proof. See Appendix A.2. □

This proposition partitions the parameter space into two regions: the invertible region in which prices fully reveal the underlying fundamental, and the non-invertible region in which they do not. In the invertible region, p^t contains the same information as the nominal aggregate

[12] The function $g(L)$ is said to have an inside root if there exists a complex number, z, inside the unit circle of the complex plane, such that $g(z) = 0$.

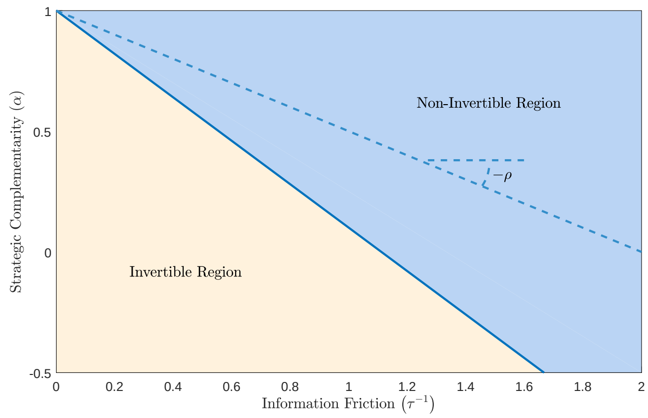


Figure 1: Two Regions of Price Informativeness

The solid line corresponds to $\rho = 0.9$ and the dashed line to $\rho = 0.5$.

demand q^t, and therefore we call this the *perfect revealing regime*. In the non-invertible region, p^t contains less information than q^t and firms are no longer able to infer the monetary shocks perfectly, which we refer to as the *confounding regime*.[13]

As shown in Figure 1, the invertible region features relatively mild degrees of informational frictions and strategic complementarity. The opposite prevails in the non-invertible region. To understand this partition of the parameter space, we first go through a more "mechanical" explanation, and then we provide a more intuitive argument. Consider the impulse response function of prices to a monetary shock, and suppose that the process of p_t is invertible. We now explore under which conditions this conjecture can be verified. In period $t = 0$, firms observe the price history except for the current one. Since it is common knowledge that the monetary shocks in the past are equal to zero, to infer the current monetary shock firms rely solely on their private signal, $x_{i,0}$. The aggregate price level in this period, p_0, is determined the same way as in a static beauty-contest game with exogenous information. In period $t = 1$, firms observe p_0 and, by invertibility, q_0 becomes common knowledge. It follows that, in periods $t \geq 1$, prices are perfectly in line with the nominal aggregate demand, $p_t = q_t = \rho^t q_0$. Taking stock, the law of motion of price is given by

$$g(L) = \underbrace{p_0}_{\text{impact effect}} + \frac{\rho L}{1 - \rho L}, \tag{8}$$

[13]The invertibility condition used here is similar to the notion of informativeness in Rondina (2008). If we instead consider an independent-value best response, we would recover the same condition obtained there.

where the first term, p_0, captures the impact effect, and the second term captures the response in the following periods when the monetary shock effectively becomes public.

The analysis above is based on the assumption that the process of p_t is invertible. To examine whether this is indeed the case, we need to check if any of the roots of $g(L)$ are inside the unit circle, which is true if and only if

$$p_0 = \frac{(1-\alpha)\tau}{1+(1-\alpha)\tau} > \frac{\rho}{1+\rho}. \tag{9}$$

That is, prices perfectly aggregate information if and only if the initial response is large enough. It is exactly when the informational friction is relatively small (large τ) and the coordination motive is relatively weak (small α) that the impact response is sizable. The former increases the responsiveness of first- and higher-order expectations, and the latter decreases the weight on generally more inertial higher-order expectations.

Intuitively, the information contained in prices ultimately comes from each individual firm's private information. The more firms rely on their private signals when making decisions, the more information can be potentially encoded in prices. With more private signal noise or with a higher degree of complementarity, firms put more weight on information in the public domain which reduces the usage of private signals. This intuition is reminiscent of the one in Amador and Weill (2010), where an increase of the public signal's precision may actually lower the information content of prices as the information contained in private signals is utilized less than a social planner would prefer.

This intuition is subject to the following caveat. Recall that, with exogenous information, independent of the informational friction and coordination motive, the price process is always invertible. With endogenous information, invertibility is obtained in a subset of the parameter space. This difference highlights that the informativeness of prices is affected not only by their overall responsiveness but also by their particular dynamic pattern. The importance of *dynamic* information aggregation also manifests itself in condition (7). With $\rho = 0$, information is always perfectly aggregated, and higher levels of ρ move the fundamental process away from a static i.i.d. and increases the non-invertible region, as can be seen in Figure 1.

Condition (7) can also be used to understand the effect of the length of a period on invertibility. When a model period is shortened, for instance, the parameters of the model need to be adjusted accordingly. First, the persistence of the fundamental, ρ, should be increased. Second, the innovations to the fundamental should be less volatile. Finally, there is less time for agents to collect information, so it is not clear whether the precision of private signals, τ, should be higher or not. According to condition (7), the equilibrium is less likely to be non-invertible if the precision increases more than proportionally to the persistence.

Proposition 3.1 already indicates that varying the degree of strategic complementarity affects the invertibility of prices. At the same time, the price informativeness χ changes with the complementarity. A higher α diverts agents from the use of private information, which tends to lower price informativeness. However, this intuition is only true in the non-invertible region, while the price informativeness χ always equals to one in the invertible region. Woodford (2003) emphasizes that higher strategic complementarity results in more inertia in inflation as higher-order expectations play a more important role. Our results imply that, when information is endogenous, there is an additional information channel through which the degree of complementarity shapes the aggregate outcome.

3.3. Price Dynamics

We have characterized the invertible region in which prices perfectly aggregate information and the corresponding price dynamics. What does the stochastic process for prices look like if the parameters are in the non-invertible region? The following proposition shows that the price dynamics becomes significantly more complex.

Proposition 3.2.

1. *In the invertible region, there exists a unique law of motion for p_t:* $p_t = q_t - \frac{1}{1+(1-\alpha)\tau}\eta_t$.
2. *In the non-invertible region, the law of motion of p_t does **not** have a finite-state representation.*

Proof. See Appendix A.3. □

In the invertible region, the law of motion for prices differs from the nominal aggregate demand only at the impact response, following an ARMA(1,1) process. To visualize the process, the red dashed line in Figure 2 plots the impulse response of prices, p_t, with $\alpha = 0.1$, which is on the edge of the invertible region (we set $\rho = 0.9$, and $\tau = 1$). Except for the first period, prices are equal to nominal aggregate demand. Varying the parameters within the invertible region only changes the initial response. Recall that real output is equal to the difference between the nominal aggregate demand and the price level, $y_t = q_t - p_t$. So, in this case, the effect of the shock on real output is only transitory. This is in line with the conventional wisdom that the effects of a monetary shock to real aggregates are transitory when prices become public.

In the non-invertible region, the law of motion for p_t does not follow any finite ARMA process. In the literature, when the number of signals is the same as the number of shocks, one can typically obtain the equilibrium law of motion with a finite-state representation (see Kasa (2000), Kasa, Walker, and Whiteman (2014), Rondina and Walker (2021) for example). In contrast, in our environment, where the fundamental is invertible and the non-invertibility of equilibrium prices is obtained endogenously when the degree of strategic complementarity is high enough, no stochastic process with a finite number of inside roots can be supported as an equilibrium. We obtain this result with a proof by contradiction: assuming the equilibrium process has a finite number of inside roots, we show that the implied actual outcome always contains more inside roots than initially conjectured. Unlike the invertible region, in the non-invertible region we cannot guarantee that there exists a unique equilibrium.[14] However, our results hold for all possible equilibria that may arise in the non-invertible region. For the parameters considered in this paper, our numerical solution always converges to the same equilibrium.

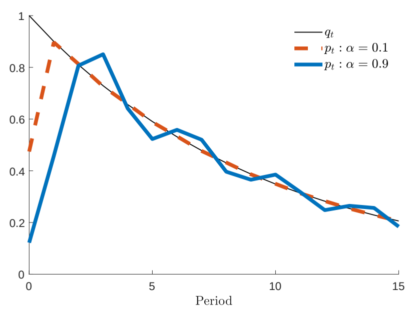


Figure 2: Response of Aggregate Price to Monetary Shock

Parameters: $\rho = 0.9$, and $\tau = 1$.

In Figure 2, the blue solid line shows the impulse response for p_t with $\alpha = 0.9$, which is in the non-invertible region. The response of p_t is initially dulled, and builds up gradually. Different from the invertible case, it does not coincide with the nominal demand, q_t, after period $t = 1$. Instead, it oscillates around q_t. This pattern is the result of the imperfect dynamic information aggregation. To understand this oscillation pattern, it is useful to consider the following two observations: First, recall from Lemma 3.1 that prices are essentially the average forecasts about the fundamental, which cannot systematically deviate from the nominal demand. For example, with exogenous information, the impulse response of price is uniformly below that of the nominal aggregate demand, and this implies a persistent underestimation of the monetary shock and past prices. However, this type of persistent underestimation of past prices does not

[14]To the best of our knowledge, the literature has not been able to give a general proof of the uniqueness result when information is endogenous and persistent.

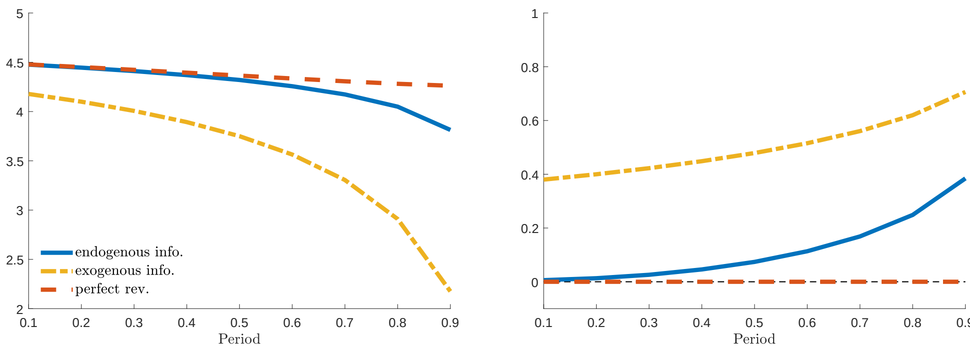
square with the fact that past prices are already part of agents' information set. Second, prices cannot be precisely equal to the nominal aggregate demand, as this would make them perfectly informative, contradicting the assumed non-invertibility of the price process. As a result, the only possible scenario is the waves of optimism and pessimism that we see here.[15]

A direct implication of Proposition 3.2 is that monetary shocks can induce persistent real output fluctuations. To allow long-lasting effects of monetary shocks on output, it is necessary that prices do not fully reveal the underlying shock. Woodford (2003) directly assumes that past prices cannot be observed, which can be rationalized by rational inattention, as in Maćkowiak and Wiederholt (2009). Another way to avoid perfect revelation is to introduce additional exogenous aggregate shocks, as in Nimark (2008) and Melosi (2017), or to add endogenous sentiment shocks, as in Acharya, Benhabib, and Huo (2021). Our results differ from those in the literature. The non-invertibility of the price process here is an endogenous equilibrium outcome, and whether it is indeed the case depends on the underlying parameters. Prices efficiently aggregate information only when individual firms' responses to their *private* signals are sufficiently strong. When such response is relatively weak, monetary shocks have persistent real effects even if past prices are perfectly observed.

Though the impulse response functions (IRF) of prices are different from the exogenous-information and the perfect-revealing cases, one may question whether the differences in conditional responses also lead to different unconditional moments. The short answer is yes. The left panel of Figure 3 compares the variance of prices for economies with: endogenous information, exogenous information, and perfect revelation of past shocks. With higher degrees of complementarity, α, agents put less weight on their exogenous private signals, which leads to a more dampened response. With perfect revelation or with endogenous information, this effect is limited as agents can still learn from the past. However, when entering the non-invertible region, the reduction of price volatility in the endogenous-information economy starts to accelerate. This effect on prices has implications for output. The right panel of Figure 3 presents the persistence of the real output. With endogenous information, the effect of monetary shock on output becomes persistent when the price becomes non-invertible. The fact that complementarities have different qualitative effects depending on its value is a feature that appears only in

[15] Due to the fact that the price process does not allow a finite-state representation, the equilibrium can only be solved numerically. We impose the price process follows a finite MA process to approximate the actual price process, and we choose the finite MA process to minimize the distance between the perceived and the actual price process.

the economy with endogenous information.



(a) variance of price p_t

(b) autocorrelation of output y_t

Figure 3: Unconditional Moments Varying (α)

Parameters: $\rho = 0.9$ and $\tau = 1.0$.

We conclude this subsection with a remark on uniqueness of equilibrium. In general, multiple equilibria may arise when information is endogenous (see Gaballo (2018) and Acharya, Benhabib, and Huo (2021) for example). In our setting, the equilibrium is unique in the invertible region, which we show by construction. Though we cannot rule out multiplicity in the non-invertible region, we think it is not likely for the following reason: the common cause of multiplicity is the strong complementarity between the responsiveness of the current action and the informativeness of endogenous signals. Since we assume that the aggregate outcome is observed with a one-period delay, the informativeness of newly arrived signals does not depend on agents' contemporaneous actions.[16] Finally, regardless of whether the equilibrium is unique or not, all the theoretical results and analyses we have derived are still valid.

3.4. Forecast Errors

Besides the different dynamics of aggregate outcomes, economies with endogenous information also generate different predictions about forecast errors relative to the other two benchmarks. Figure 4 illustrates this point by plotting the responses of one-period ahead forecast errors about the price level, with the variance of innovations to the fundamental chosen such that the peaks of responses are identical. First consider the case with endogenous information.

[16] If there is an initial period, it follows that the equilibrium is always unique. Moreover, though this is certainly not definitive, our numerical algorithm always yields a unique solution.

When the price is non-invertible, observing past prices does not reveal all information and forecast errors cannot return to zero immediately. At the same time, agents' forecasts have to be consistent with observed past prices, which prevents forecast errors from staying positive indefinitely: agents persistently under-estimating prices would contradict the continuous observation of past realizations. As a result, forecast errors are positive initially and oscillate around zero afterwards.

In contrast, with exogenous information, the forecast error is persistent and declines monotonically. Since information only comes from exogenous noisy signals, persistent under-estimation is consistent with optimal forecasts. With perfect revelation of past shocks, the forecast error disappears immediately after one period, as past aggregate shocks becomes common information.

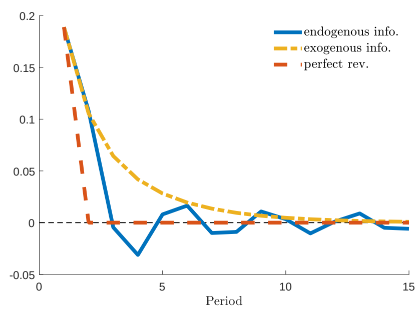


Figure 4: Forecast errors for different information structures

Parameters: $\rho = 0.9$, $\alpha = 0.9$, and $\tau = 1$. We adjust the variance of the innovation to the fundamental, η_t, so that all three models have the same maximum forecast error level—$\sigma_\eta = 1$ for the endogenous information model, $\sigma_\eta = 1.75$ for the exogenous information model, and $\sigma_\eta = 0.42$ for the perfect revealing model.

The pattern of forecast errors also helps connect with survey evidence on expectations. With non-invertible price dynamics, the forecasts underreact to the aggregate shocks on average, consistent with Coibion and Gorodnichenko (2015). Under-reaction is significantly less pronounced when past shocks are common information. The sign-switching pattern of forecast errors is also consistent the evidence documented in Angeletos, Huo, and Sastry (2020) and Kucinskas and Peters (2018). Moreover, Kohlhas and Walther (2021) show that the correlation between forecast errors and current outcomes tends to be negative. Oscillatory forecast errors also help lower this correlation. We explore the implications of endogenous information aggregation for these moments further in Section 4. Finally, adding noise to price observation lowers the frequency and amplitude of these oscillations, which may be helpful in matching the data.

3.5. Beyond Perfect Price Observation

In this subsection, we explore to what extent our previous results remain relevant if prices are not perfectly observed. Imperfect observation could arise when prices are recorded with measurement errors, or if firms perceive published prices in an imperfect way due to costly contemplation. We show that the exogenous and endogenous information structures serve as two useful benchmarks: the outcomes with noisy price observation live between these two extremes. This observation also extends to setups with multiple aggregate shocks.

3.5.1. Noisy Price Observation

Suppose firms observe past prices with idiosyncratic noise ν_{it},

$$z_{it} = p_{t-1} + \nu_{it}, \quad \nu_{it} \sim \mathcal{N}(0, \kappa^{-1}). \tag{10}$$

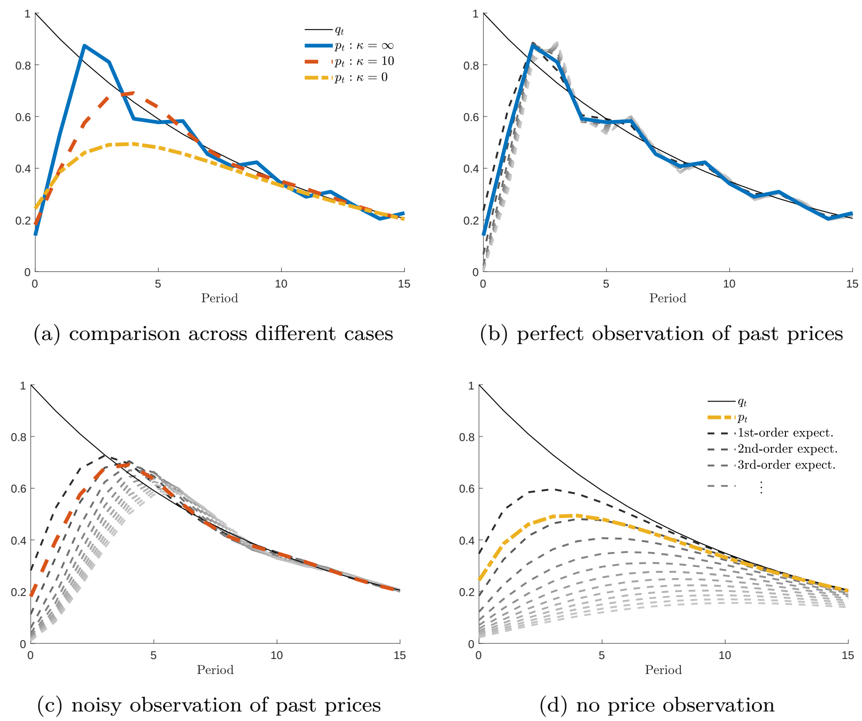
The information set for firm i, then, becomes $\mathcal{I}_{it} = \{x_i^t, z_i^t\}$. This information structure nests the two benchmarks we have studied: by setting κ to infinity, we return to the case where prices are perfectly observed; by setting κ to zero, we obtain the other extreme where firms only have access to exogenous information, $\{x_i^t\}$. In the intermediate range, information is still endogenous as the informativeness of the price signal, z_{it}, is determined endogenously in equilibrium.[17]

We first show how the price dynamics changes when we vary the noise to the price observation. In Panel (b) of Figure 5, the blue solid line corresponds to the case in which prices are perfectly observed, $\kappa \to \infty$, and the parameters lie in the non-invertible region. When the magnitude of the private noise is positive but relatively small, by continuity, the price dynamics is similar to the limiting case. The red dashed line in Panel (c) confirms this logic: with $\kappa = 10$, it resembles the pattern of the blue solid line though the oscillation around the fundamental is less pronounced. When $\kappa = 0$ (the exogenous information case), there is no oscillation anymore. The yellow line in Panel (d) is always below the fundamental.

3.5.2. Multiple Aggregate Shocks

So far, independent of whether past prices are perfectly observed or not, the price dynamics is driven by a single aggregate shock. In reality, there can be multiple underlying shocks that determine aggregate outcomes. Similar to the intuition developed earlier, prices continue to exhibit the type of dynamics described in our baseline model as long as the magnitude of

[17] To solve the intermediate case, one has to resort to numerical methods as, generally, no finite-state solution is available. See Huo and Takayama (2022) for a more detailed discussion.



(a) comparison across different cases

(b) perfect observation of past prices

(c) noisy observation of past prices

(d) no price observation

Figure 5: Response of Aggregate Prices Varying Precision of Price Observation (κ)

Parameters: $\rho = 0.9$, $\alpha = 0.5$, and $\tau = 0.25$. In panel (b), $\kappa = \infty$; in panel (c), $\kappa = 10$; in panel (d), $\kappa = 0$.

the "additional" shocks are not too large. That is, the results in our baseline specification serve, again, as a particular benchmark. The magnitude of deviations from this benchmark is a quantitative question and depends on the particular application one is considering. In Appendix B, we explore the information structure where the exogenous signal contains an aggregate noise shock, and the intuition developed earlier is confirmed in numerical examples.

3.6. Extensions

Though the baseline model developed in this section is rather special, the main insights we obtained extend to much more general environments. Our main results extend to significantly more involved information structures, allowing for fundamentals that follow any stochastic process, and multiple public and private signals with noises that also follow arbitrary processes. In addition, we extend the results to environments with more sophisticated linkages between agents featuring forward and backward complementarities, and multivariate systems that can be viewed as a network game with incomplete information. We delegate a detailed discussion of these results to Appendix C.

4. Applications in New Keynesian Models

In this section, we extend our results to a New Keynesian model with incomplete information. The main insights of Section 2 are developed in static beauty contests, while New Keynesian models are forward-looking in nature. We therefore leverage the additional theoretical results from Appendix C. We start with an analysis that treats the demand block and the supply block independently, and focus on how the key parameters such as the price stickiness and the MPC, which governs the GE feedback effects, also modify how much information is aggregated. We then unify the two blocks to explore the implications of the aggressiveness of the Taylor rule on aggregate outcomes. Finally, we show the effects the endogenous information in a calibrated model disciplined by the survey evidence on expectations.

4.1. Effect of price stickiness and MPCs

First consider the two building blocks of the New Keynesian model, the Dynamic IS curve, and the New Keynesian Philips curve (NKPC). Relaxing the common-knowledge foundations of the New Keynesian model along the lines of Angeletos and Lian (2018) yields[18]

$$\pi_{it} = \kappa\,\mathbb{E}_{it}[mc_t] + (1-\theta)\,\mathbb{E}_{it}[\pi_t] + \delta\theta\,\mathbb{E}_{it}[\pi_{it+1}], \tag{11}$$

$$c_{it} = -\varsigma(1-\texttt{mpc})\,\mathbb{E}_{it}[r_t] + \texttt{mpc}\,\mathbb{E}_{it}[c_t] + (1-\texttt{mpc})\,\mathbb{E}_{it}[c_{i,t+1}]. \tag{12}$$

In the NKPC, mc_t is the real marginal cost, θ is the Calvo parameter, and δ is the discount rate. In the dynamic IS curve, r_t is the real rate, ς is the intertemporal elasticity of substitution, $\texttt{mpc}$ is the marginal propensity to consume. With complete information, these two conditions reduce to their familiar textbook versions. Crucially, the degree of price stickiness, θ, and the MPC control the degree of static strategic complementarity in firms' pricing decision and consumers' saving-consumption decision respectively.

Here, we treat the marginal cost, mc_t, and the real interest rate, r_t, as exogenous fundamentals, and therefore the supply block (11) and the demand block (12) do not interact with each other and can be treated separately, as in Nimark (2008) and Angeletos and Huo (2021). As in our baseline specification, we assume that the fundamentals mc_t and r_t follow AR(1) processes. Every period, firms and consumers observe an exogenous signal about the fundamental with private noises, and the past realization of π_t and c_t, respectively. The following proposition connects the deep parameters with the invertibility of aggregate outcomes.

[18] Appendix D presents a derivation of these equations together with the proof of Corollary 4.1.

Proposition 4.1. *For a fixed level of private noise, inflation is noninvertible when prices are sufficiently flexible (lower θ), and aggregate consumption is noninvertible when the MPC is sufficiently high (higher $\mathtt{mpc}$).*

Proof. See Appendix D. □

The underlying logic for this result can be understood as follows: a lower θ implies more frequent price adjustments. As a result, for the firms that can adjust their prices, there is a greater need to worry about other firms' price-setting decisions. A higher MPC makes consumption more sensitive to income changes, which leads to a stronger dependence on aggregate demand. Both of these effects map to a higher degree of strategic complementarity and therefore make the economy more likely to be non-invertible—see Appendix C for a more detailed analysis.

4.2. Role of the aggressiveness of the nominal interest rate response

In the previous example, we treat inflation and output separately. In this section, we show that the main insights extend to the case where the supply and demand blocks of the economy interact with each other. Particularly, we show that when the central bank follows a standard Taylor rule, a less aggressive response of the nominal interest rate to inflation tends to reduce the amount of information aggregated by output and inflation, yielding a non-invertible equilibrium.

We extend the model in the following way,

$$\pi_{it} = \kappa \, \mathbb{E}_{it}[y_t + \xi_t^s] + (1-\theta)\, \mathbb{E}_{it}[\pi_t] + \delta\theta \, \mathbb{E}_{it}[\pi_{it+1}], \tag{13}$$

$$c_{it} = -\varsigma(1-\mathtt{mpc})\, \mathbb{E}_{it}[i_t - \pi_{t+1} + \xi_t^d] + \mathtt{mpc}\, \mathbb{E}_{it}[c_t] + (1-\mathtt{mpc})\, \mathbb{E}_{it}[c_{i,t+1}], \tag{14}$$

$$y_t = c_t, \tag{15}$$

$$i_t = \phi_\pi \pi_t. \tag{16}$$

The first equation is the NKPC, modified to allow a real marginal cost proportional to aggregate output, and subject to a cost-push shock ξ^s. The second equation is the dynamic IS curve with the real interest rate replaced by the difference between nominal interest rates and expected future inflation, and augmented with a preference shock ξ_t^d. The third equation is a market clearing condition (useful for the next subsection) and the forth equation is the Taylor rule for the nominal interest rate, where ϕ_π controls how aggressive monetary policy reacts to inflation.

In terms of information, we allow individual firms and consumers to observe private signals about the demand and supply shocks, $x_{i,t}^s = \xi_t^s + u_{i,t}^s$, $x_{i,t}^d = \xi_t^d + u_{i,t}^d$, as well as past inflation and output realizations. To simplify the analysis, we assume that both ξ_t^s and ξ_t^d follows an AR(1) process with persistence ρ, and that $u_{i,t}^s$ and $u_{i,t}^d$ are both i.i.d shocks with variance τ^{-1}.[19]

The system (13)-(16) effectively consists of a forward-looking network game. As a result, the strength of GE effects, or the degree strategic complementarity, no longer depends on a single scalar but on all the relevant structural parameters that control within-group and cross-group dependences. Here, we highlight the role of ϕ_π. When the nominal interest rate is passive (such as in the zero-lower bound), higher inflation induces higher demand, which further pushes up marginal costs and inflation. A higher ϕ_π suppresses aggregate demand's response and mutes this reinforcing mechanism, which implies a higher degree of strategic *substitutability*.

With perfect information, varying ϕ_π only matters for the magnitude of the responses of output and inflation to shocks. With incomplete information and the observation of endogenous outcomes, varying ϕ_π also changes the information content of c_t and π_t. Since a higher ϕ_π implies higher strategic substitutability, it makes agents rely more on their private signals. Thus, the economy is more likely to be invertible in the sense that, by observing $\{c^t, \pi^t\}$, one can perfectly infer the demand and supply shocks, i.e.,[20]

$$\mathbb{E}[\xi_t^s | c^t, \pi^t] = \xi_t^s, \quad \text{and} \quad \mathbb{E}[\xi_t^d | c^t, \pi^t] = \xi_t^d.$$

This argument is formalized in the following proposition.

Proposition 4.2. *The outcomes $\{\pi^t, c^t\}$ perfectly aggregate information only if ϕ_π exceeds the threshold Φ given by*

$$\Phi = \begin{cases} \frac{\rho^2 - \theta(1-\mathsf{mpc})\tau^2}{\kappa\varsigma(1-\mathsf{mpc})\tau^2}, & \textit{when } (1-\theta-\mathsf{mpc})^2 < 4(1-\mathsf{mpc})(\theta+\kappa\varsigma\phi_\pi), \\ \\ \frac{\tau\rho(1+\theta-\mathsf{mpc})-\rho^2-\theta(1-\mathsf{mpc})\tau^2}{\kappa\varsigma(1-\mathsf{mpc})\tau^2}, & \textit{otherwise.} \end{cases} \tag{17}$$

Proof. See Appendix A.4. □

In the invertible region, inflation and output differ from their perfect information counterparts only in the initial period. In contrast, in the non-invertible region, endogenous outcomes

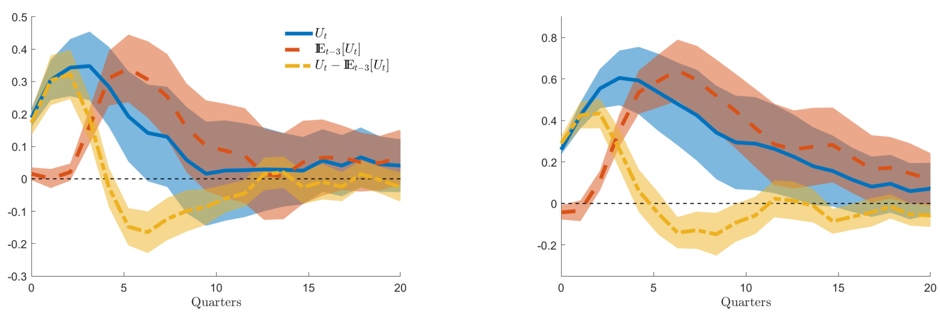
[19] The main conclusion of our results on the comparative statics on ϕ_π does not depend on this symmetry assumption.

[20] Technically, invertibility requires the equilibrium process $\begin{bmatrix} c_t \\ \pi_t \end{bmatrix} = \mathbf{g}(L) \begin{bmatrix} \eta_t^s \\ \eta_t^d \end{bmatrix}$ to satisfy the condition that $\det[\mathbf{g}(L)]$ does not contain any inside root.

display an early under-reaction and an oscillatory pattern later on. This suggests that the central bank's policy rule matters not only for the magnitude of the response of aggregate outcomes to shocks but also disciplines their entire dynamic patterns.

4.3. Quantitative Assessment

In this subsection, we connect the model studied in subsection 4.2 to data on expectations. We first document some salient features of the conditional responses of average forecasts from the Survey of Professional Forecasters and the Blue Chip Economic Indicators.[21] Following Angeletos, Huo, and Sastry (2020), we interpret the negative of output, y_t, as the quarterly rate of unemployment, since the two objects are closely related in theory and in the data. In the main text, we focus on the response of y_t to a demand shock, ξ_t^d. Relative to the system (13)-(16), we make an additional assumption that firms are not subject to informational frictions, and therefore a standard Phillips curve operates: $\pi_t = \frac{\kappa}{\theta} y_t + \delta \mathbb{E}_t[\pi_{t+1}]$. In Appendix E.2, we present analogous results for the response of the inflation rate to a supply shock ξ_t^s.



(a) unemployment SPF (b) unemployment Blue Chip

Figure 6: Responses of Unemployment Rate to a Demand Shock

The solid blue lines in Figure 6 displays, how the unemployment rate responds to the main business cycle shock (Angeletos, Collard, and Dellas, 2020) in the data. This shock is identified by maximizing the volatility of unemployment at the business-cycle frequency, and accounts for the bulk of short-run fluctuations of main real aggregate variables. The dashed red lines and the dotted yellow lines correspond to the three-quarter ahead forecasts and forecast errors,

[21]The sample period is from 1968 q4 to 2017 q4. We use the median forecasts instead of the average forecasts to avoid the influence outliers in a small sample, and employ the initial realise of the data on outcomes to construct the forecast errors.

respectively. Three features are salient: (1) the outcomes respond sluggishly and peak about one year after the shock; (2) the response of aggregate forecasts are dampened on impact, implying that agents underestimate outcomes initially; and (3) the aggregate forecasts exceed the outcomes later on, resulting in a sign-switching pattern for forecast errors.

In our baseline model with endogenous information aggregation, agents observe the past aggregate outcome, y_{t-1}, and receive an exogenous signal, x_{it}, about the economic fundamental,

$$x_{it} = \xi_t^d + u_{it}, \quad u_{it} \sim \mathcal{N}(0, \tau^{-1}).$$

For comparison, we also consider three alternative information structures: (1) exogenous information, in which agents do not observe the endogenous outcome y_{t-1}; (2) perfect revelation of past shocks, so agents directly observe ξ_{t-1}^d every period; and (3) perfect information.

4.3.1. Calibration

To quantify the role of endogenous information aggregation, we set economic parameters to numbers commonly used in the literature, and calibrate the remaining parameters that are uniquely related to our model economy by targeting the responses of the outcome and aggregate forecasts in the SPF.[22] On the supply side, we choose the Calvo probability θ to be 0.45, so that the average price duration is about 2 quarters, and the discount factor $\delta = 0.99$. The slope of the Phillips curve is set to $\kappa = 0.05$. There are different estimates for this parameter, so we check that quantitative results are robust to its choice (we obtain comparable results with $\kappa = 0.2$). On the demand side, we set the intertemporal elasticity of substitution ς to 1, and the MPC to 0.15, which lies in the middle of various estimates in the literature. We then calibrate the persistence of the demand shock, $\rho = 0.78$, the standard deviation of its innovation, $\sigma_\eta = 0.58$, and the precision of the exogenous signal, $\tau = 0.02$. Table 1 summarizes the parameter values. In the economy that perfectly reveals past information, we maintain the same precision of the exogenous signal for comparison. In the economy with only exogenous information, we re-calibrate the precision so that it obtains the same CG regression coefficient (Coibion and Gorodnichenko, 2015) for 3-period ahead forecast as in the model with endogenous information ($\tau = 1.02$).

[22] We assign equal weights to the responses of the outcome and the forecasts in the first 20 periods.

Table 1: Model Parameters

Parameter	Description	Value
θ	Calvo probability	0.45
κ	slope of Phillips curve	0.05
δ	discount factor	0.99
mpc	marginal propensity to consume	0.15
ς	intertemporal elasticity of substitution	1.00
ϕ_π	policy rule slope	1.50
ρ	persistent of demand shock	0.77
τ	precision of exogenous signals	0.02
σ_η	s.d. of innovation to demand shock	0.58

4.3.2. Conditional Moments: Model Impulse Responses

Figure 7 compares the predictions of the model about outcomes and average forecasts under different information structures. We start with perfect information at the top left panel. In this case, the outcome y_t is simply proportional to the demand shock ξ_t^d, and the initial sluggishness of its empirical counterpart is not present. The forecast error also returns to zero without any delay. The top right panel displays the results when past shocks are perfectly revealed. Relative to the perfect information results, the initial response of output is dampened due to noisy information, but it coincides with the perfect-information outcome starting from the next period. These two scenarios are clearly at odds with the data. The bottom left panel displays the result with exogenous noisy information. Both the outcome and the forecasts are dampened initially, and forecast errors gradually and monotonically returns to zero.

Now turn to the bottom right panel which presents the results with endogenous information aggregation. Our calibration implies that the output process is non-invertible. As a result, observing past aggregate outcomes only provide partial information about past shocks. Relative to the perfect revealing case, the response of output in the first year is significantly more dampened, and the hump-shaped response that follows is consistent with the data. Compared with the exogenous information case, the forecast error does not always stay positive but switches signs in period 6. This delayed overshooting is qualitatively consistent with the data, while the magnitude of the overshooting is smaller: the trough of the forecast error is about 50% of the one observed in the data. Angeletos, Huo, and Sastry (2020) match the pattern of forecast

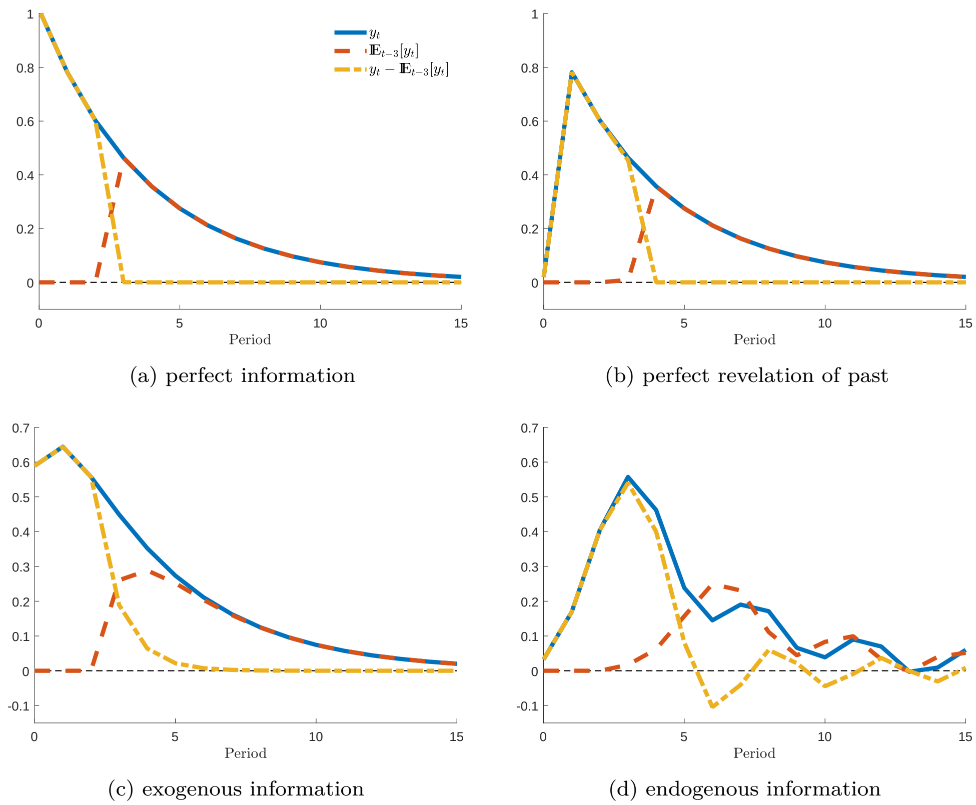


Figure 7: Responses of Output and Forecast

errors by imposing extrapolative beliefs, while agents form rational expectations in our model economy.

Remark. The VAR procedure used in Angeletos, Collard, and Dellas (2020) to generate the IRFs in Figure 6 assumes that the structural representation of the data generating process is invertible, while the output process in our model economy is non-invertible.[23] It is, therefore, not clear in principle if it is valid to compare these IRFs. To address this, in Appendix E.1, we compare the model-generated IRFs with the ones obtained by estimating a VAR, with the same identifying assumption used in Angeletos, Collard, and Dellas (2020), on simulated data from our model. The IRFs are reassuringly quite similar. In addition, though the estimated multivariate system in Angeletos, Collard, and Dellas (2020) is assumed to be invertible, this

[23]This invertibility requirement of VARs has been explored in Lippi and Reichlin (1994), Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007), and Leeper, Walker, and Yang (2013) among others.

does not exclude the possibility that the conditional response of unemployment to the main-business-cycle shock is non-invertible, which is the case our model focuses on. Finally, the unconditional moments in the next subsection are independent from this issue.

4.3.3. Unconditional Moments

In addition to the conditional responses, we also explore the effects of endogenous information on unconditional moments. Particularly, we consider the following two regressions

$$
\begin{aligned}
y_{t+k} - \overline{\mathbb{E}}_t[y_{t+k}] &= \beta_{CG}\left(\overline{\mathbb{E}}_t[y_{t+k}] - \overline{\mathbb{E}}_{t-1}[y_{t+k}]\right) + v_{t+k,t}, \\
y_{t+k} - \overline{\mathbb{E}}_t[y_{t+k}] &= \beta_{KW} y_t + v_{t+k,t}.
\end{aligned}
$$

The first regression is proposed by Coibion and Gorodnichenko (2015) and the second one is proposed by Kohlhas and Walther (2021). These two regression coefficients provide a diagnostic test for the deviation from the full-information rational expectations benchmark.

Table 2 presents these coefficients calculated under different information structures and their data counterparts. Though not directly targeted, with endogenous information, the implied β_{CG} regressions coefficients at different forecasting horizons are broadly consistent with their data counterparts. The positive β_{CG} coefficients point to under-reaction of aggregate forecasts. In contrast, with perfect revelation of past shocks, the implied β_{CG} coefficients are much closer to zero, indicating insufficient under-reaction of aggregate forecasts. We have chosen the precision of the signal in the economy with exogenous information such that the β_{CG} regression coefficient is the same as the one with endogenous information for $k = 3$. Interestingly, as forecasting horizon shortens, the regression coefficients decrease much slower with exogenous information.

Table 2: CG and KW Regressions at Different Horizons

	CG (2015) regression			KW (2021) regression		
	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
Perfect Revelation	0.03	0.03	0.04	0.01	0.01	0.01
Exogenous Information	0.70	0.72	0.73	0.16	0.14	0.11
Endogenous Information	0.25	0.47	0.73	0.01	0.02	0.01
Data	0.38	0.60	0.74	-0.00	-0.02	-0.06

The β_{KW} coefficients tend to be negative in the data. Through the lens of our model, a negative β_{KW} coefficient requires a negative correlation between the impulse responses of forecast errors and outcomes. None of the information structures considered so far can generate such negative coefficient. Nevertheless, with endogenous information the out-of-sync oscillatory nature of forecast errors and outcomes brings this correlation effectively to zero, approaching the data. Notice that with exogenous information, the β_{KW} coefficients are significantly higher than the ones with endogenous information.

5. Conclusion

We show that, with dispersed information, even when past aggregate outcomes are perfectly observed, the underlying states may not be fully revealed. The extent to which endogenous outcomes help aggregate information is increasing in the precision of private signals about economic fundamentals and is decreasing in the strength of general equilibrium feedback effects. With imperfect information aggregation, the equilibrium dynamics oscillate around the economic fundamental, and the forecast errors display an initial under-reaction and a delayed over-reaction which is consistent with the empirical evidence in Angeletos, Huo, and Sastry (2020). Together with the solution to the exogenous information environment, our results provide a second benchmark to characterize properties of equilibrium under incomplete information.

When the theory is applied to familiar macro models, it leads to several applied implications. In monetary models à la Woodford (2003), the GE feedback effect comes from firms' pricing complementarity. When it is strong enough, monetary shocks can have persistent effects even when firms observe past prices. In standard NKPC models, a less aggressive monetary policy induces a stronger GE feedback effect between inflation and output, and can result in hump-shaped and oscillatory responses of aggregate variables.

Left outside this paper is the extension beyond the standard linear-Gaussian framework. In an asset pricing setting, Albagli, Hellwig, and Tsyvinski (2015) allow a much more flexible payoff structure and shock distribution. Hassan and Mertens (2015) also incorporate dispersed information into a non-linear DSGE model, and quantify the informational role of stock prices. These works allow agents to observe past shocks, and therefore the learning is essentially static. It would be interesting to explore the extent to which agents can learn from the past in a non-linear environment.

References

Acharya, S., J. Benhabib, and Z. Huo (2021): "The anatomy of sentiment-driven fluctuations," *Journal of Economic Theory*, 195, 105280.

Adam, K. (2007): "Optimal monetary policy with imperfect common knowledge," *Journal of Monetary Economics*, 54(2), 267–301.

Albagli, E., C. Hellwig, and A. Tsyvinski (2015): "A Theory of Asset Prices based on Heterogeneous Information," Discussion paper, Yale Mimeo.

Amador, M., and P.-O. Weill (2010): "Learning from Prices: Public Communication and Welfare," *Journal of Political Economy*, 118(5), 866 – 907.

——— (2012): "Learning from Private and Public Observations of Others Actions," *Journal of Economic Theory*, 147(3), 910–940.

Angeletos, G.-M., F. Collard, and H. Dellas (2020): "Business-cycle anatomy," *American Economic Review*, 110(10), 3030–70.

Angeletos, G.-M., and Z. Huo (2021): "Myopia and anchoring," *American Economic Review*, 111(4), 1166–1200.

Angeletos, G.-M., Z. Huo, and K. Sastry (2020): "Imperfect Macroeconomic Expectations: Evidence and Theory," in *NBER Macroeconomics Annual 2020, volume 35*, NBER Chapters. National Bureau of Economic Research, Inc.

Angeletos, G.-M., L. Iovino, and L. Jennifer (2020): "Learning over the business cycle: Policy implications," *Journal of Economic Theory*, 190, 105115.

Angeletos, G.-M., and J. La'O (2010): "Noisy Business Cycles," in *NBER Macroeconomics Annual 2009, Volume 24*, pp. 319–378. University of Chicago Press.

Angeletos, G.-M., and C. Lian (2018): "Forward guidance without common knowledge," *American Economic Review*, 108(9), 2477–2512.

Angeletos, G.-M., and A. Pavan (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75(4), 1103–1142.

——— (2009): "Policy with Dispersed Information," *Journal of the European Economic Association*, 7(1), 11–60.

Bacchetta, P., and E. V. Wincoop (2006): "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review*, 96(3), 552–576.

Baxter, B., L. Graham, and S. Wright (2011): "Invertible and non-invertible information sets in linear rational expectations models," *Journal of Economic Dynamics and Control*, 35(3), 295–311.

Chahrour, R., and G. Gaballo (2021): "Learning from house prices: Amplification and business fluctuations," *The Review of Economic Studies*, 88(4), 1720–1759.

Coibion, O., and Y. Gorodnichenko (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, 105(8), 2644–78.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007): "ABCs (and Ds) of understanding VARs," *American economic review*, 97(3), 1021–1026.

Gaballo, G. (2018): "Price Dispersion, Private Uncertainty, and Endogenous Nominal Rigidities," *Review of Economic Studies*, 85(2), 1070–1110.

Graham, L., and S. Wright (2010): "Information, heterogeneity and market incompleteness," *Journal of Monetary Economics*, 57(2), 164–174.

Grossman, S. J. (1976): "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information," *Journal of Finance*, 31(2), 573–585.

Han, Z., X. Ma, and R. Mao (2022): "The role of dispersed information in inflation and inflation expectations," *Review of Economic Dynamics*.

Hassan, T. A., and T. M. Mertens (2015): "Information Aggregation in a Dynamic Stochastic General Equilibrium Model," *NBER Macroeconomics Annual*, 29(1), 159–207.

Hayek, F. A. (1945): "The Use of Knowledge in Society," *The American Economic Review*, 35(4), 519–530.

Hellwig, C., and V. Venkateswaran (2015): "Dispersed Information, Sticky Prices and Monetary Business Cycles: A Hayekian Perspective," Discussion paper.

Hellwig, M. F. (1980): "On the aggregation of information in competitive markets," *Journal of Economic Theory*, 22(3), 477–498.

Huo, Z., and M. Pedroni (2020): "A Single-Judge Solution to Beauty Contests," *American Economic Review*, 110(2), 526–68.

Huo, Z., and N. Takayama (2022): "Rational Expectations Models with Higher Order Beliefs," Manuscript, Yale University.

Kasa, K. (2000): "Forecasting the Forecasts of Others in the Frequency Domain," *Review of Economic Dynamics*, 3(4), 726–756.

Kasa, K., T. B. Walker, and C. H. Whiteman (2014): "Heterogeneous Beliefs and Tests of Present Value Models," *Review of Economic Studies*, 81(3), 1137–1163.

Kohlhas, A. N., and A. Walther (2021): "Asymmetric attention," *American Economic Review*, 111(9), 2879–2925.

Kucinskas, S., and F. Peters (2018): "Measuring Biases in Expectation Formation," Tinbergen Institute Discussion Papers 18-058/IV, Tinbergen Institute.

Leeper, E. M., T. B. Walker, and S.-C. S. Yang (2013): "Fiscal foresight and information flows," *Econometrica*, 81(3), 1115–1145.

Lippi, M., and L. Reichlin (1994): "VAR analysis, nonfundamental representations, Blaschke matrices," *Journal of Econometrics*, 63(1), 307–325.

Lucas, R. J. (1972): "Expectations and the neutrality of money," *Journal of Economic Theory*, 4(2), 103–124.

Maćkowiak, B., and M. Wiederholt (2009): "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, 99(3).

Melosi, L. (2017): "Signalling effects of monetary policy," *The Review of Economic Studies*, 84(2), 853–884.

Messner, S., and X. Vives (2005): "Allocative and Productive Efficiency in REE with Asymmetric Information," Discussion paper, IESE mimeo.

Morris, S., and H. S. Shin (2002): "Social Value of Public Information," *American Economic Review*, 92(5), 1521–1534.

Nimark, K. (2008): "Monetary policy with signal extraction from the bond market," *Journal of Monetary Economics*, 55(8), 1389–1400.

Nimark, K. P. (2014): “Man-Bites-Dog Business Cycles,” *American Economic Review*, 104(8), 2320–67.

Rondina, G. (2008): “Incomplete Information and Informative Pricing: Theory and Application,” 2008 Meeting Papers 981, Society for Economic Dynamics.

Rondina, G., and T. B. Walker (2021): “Confounding dynamics,” *Journal of Economic Theory*, 196, 105251.

Sargent, T. J. (1991): “Equilibrium with signal extraction from endogenous variables,” *Journal of Economic Dynamics and Control*, 15(2), 245–273.

Sims, C. A. (2003): “Implications of Rational Inattention,” *Journal of Monetary Economics*, 50(3), 665–690.

Townsend, R. M. (1983): “Forecasting the Forecasts of Others,” *Journal of Political Economy*, 91(4), 546–88.

Vives, X. (2014): “On The Possibility Of Informationally Efficient Markets,” *Journal of the European Economic Association*, 12(5), 1200–1239.

——— (2017): “Endogenous Public Information and Welfare in Market Games,” *Review of Economic Studies*, 84(2), 935–963.

Woodford, M. (2003): “Imperfect Common Knowledge and the Effects of Monetary Policy,” *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, p. 25.