Sticky Wage Models and Labor Supply Constraints

By Zhen Huo and José-Víctor Ríos-Rull*

In sticky wages models (either à la Calvo or à la Rotemberg), labor is solely determined by the demand side. However, a change of circumstances may make labor demand higher than agents' willingness to work. We find that workers are required to work against their will between 15 percent and 30 percent of the time (with 5 percent wage markup, less with higher markups and in Rotemberg models). Estimating models with the minimum of the demand and supply of labor instead of the demand-determined quantity yields different and unappealing properties. Hence, special attention should be paid to possible violations of the labor supply constraint.
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In New Keynesian models with sticky wages à la Calvo or à la Rotemberg, the quantity of labor is solely determined by the demand side, implicitly assuming that households are always willing to work at whatever wage rate is specified. This assumption can be justified if the amount of labor is not larger than what agents are willing to work, or what we refer to as the *labor supply constraint* is not violated. Unions with monopsony power set wages above agents' marginal willingness to work, which provides a cushion that accommodates the effects of various shocks on the demand and supply of labor. If the shocks to the economy are not too large, the cushion is sufficient to guarantee that households happily accommodate the quantity of labor required. In this paper we document that the cushion may be too small in popular DSGE models: demand-determined labor often implies that some of the labor is provided against the will of the workers, a violation of the principle of voluntary exchange.

What is the natural alternative to the violation of the labor supply constraint? Here we have taken the strict position of staying within the two types of models that we explore, rather than proposing a change of model. In the Calvo model trade occurs at non-market clearing prices. For these type of environments, Drèze (1975), following the notion of disequilibrium modeling of Barro and Grossman (1971) and Malinvaud (1977), posed that the amount traded is the minimum of the quantities supplied and demanded and that the agents are aware of the

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limitation in the availability of the trades.[1] The Drèze equilibrium provides an outcome that satisfies individual rationality without changing any feature of the environment and hence we think that it is the natural equilibrium concept when wages are deemed to be fixed, as they are in sticky wage à la Calvo models. For the Rotemberg model, we think that the obvious alternative to ignoring the labor supply constraint is to let the unions internalize the constraint when they set prices, automatically inducing that the constraint is not violated.

We start our analysis by first substituting the demand-determined quantity of labor that the log-linearization procedure delivers with the minimum of the quantity of labor demanded and the quantity that agents would like to work. We refer to this quantity as *voluntary ex-post aggregate labor or just* ex-post labor.[2] The comparison between the two series tells us whether some agents are working against their will in demand-determined allocations. The ex-post labor is not part of an equilibrium (Drèze or otherwise), since agents made their decisions based on the demand-determined quantity of labor, but it does give us a preliminary account of the extent to which the demand-determined allocation is consistent with agents not working against their will.

We carry out this comparison in two of the most standard models of the New Keynesian literature: Altig et al. (2011) and Smets and Wouters (2007). With Calvo pricing we find that the properties of the two measures of labor are quite different: In the Altig et al. (2011) model, the fraction of workers with labor supply constraint being violated varies from 19 percent to almost zero as the wage markup moves from 5 percent to 25 percent. The variance of labor shrinks by 15 percent for a wage markup of 15 percent when we move from the demand-determined to the ex-post quantity of labor. In the Smets and Wouters (2007) model, wage markup shocks are important in accounting for wage and labor movements, but their magnitude and structural interpretation are controversial (as we discuss below). Therefore, we look at versions with and without these shocks. With wage markup shocks, the fraction of workers that have their labor supply constraint violated varies from 45 percent to 16 percent as the wage markup moves from 5 percent to 25 percent. In expansions, ex-post labor shrinks so much so that the volatility actually becomes larger than that in the demand-determined case. With a 15 percent wage markup, the variance increases by 50 percent. When wage markup shocks are excluded, the fraction of workers that have their labor supply constraint violated varies from 32 percent to 2 percent as the wage markup goes from 5 percent to 25 percent. The variance of labor is 15 percent smaller in the demand-determined model for a wage markup of 15 percent.

[1] This requires that all firms internalize that they are treated equally when facing a limited labor supply. We can see this as the result of assuming that firms send bids and that the available workers are equally distributed between all firms. That firms understand this is consistent with the model. An alternative that would add a lot of complexity without any substance is to pose a randomization mechanism.

[2] The calculation of ex-post labor is not a trivial endeavor: Calvo pricing implies that many different wages coexist at any point in time depending on the exact period when the wage was last set, and there is a different quantity of labor associated to each one of those wages.

Within the Rotemberg wage setting, the magnitude of the differences is still noticeable, but somewhat smaller: in the Altig et al. (2011) model, the frequency of the labor supply constraint violation is 5 percent with a 5 percent wage markup (and close to zero for larger markups), while the variance of labor is 10 percent smaller for the ex-post labor. In the Smets and Wouters (2007) model with wage markup shocks, the frequency of the violation of the labor supply constraint is 36 percent with a 5 percent wage markup and still 6 percent with a 25 percent markup. The variance of labor is also smaller for the ex-post labor, between 15 percent and 20 percent smaller, for a 5 percent wage markup depending on whether we include all shocks or not.

Ensuring that the labor supply constraint is not violated, either by using the Drèze equilibrium in the Calvo model or by having the unions internalize the constraint in the Rotemberg model, has severe computational challenges.[3] For this reason we propose an equilibrium approximation strategy that we use in both types of models. We verify in a simpler class of New Keynesian models (wage rigidity either a la Taylor or la Rotemberg and no bells and whistles in other dimensions) that the non-approximated equilibria and the approximated equilibria are very similar. This gives us confidence on the reliability of the findings that we obtain with the approximated equilibria in medium scale DSGE models with Calvo and Rotemberg prices.

We proceed to estimate a version of the Altig et al. (2011) economy with Calvo pricing under the approximated Drèze equilibrium, and we find that the economic properties are very different in the demand-determined solution than when the labor supply constraint is required to be satisfied. The relative importance of the various shocks changes dramatically, with neutral technology shocks accounting for 71 percent of the variance of labor instead of the 13 percent obtained with the demand-determined solution. Moreover, neutral technology shocks tend to have larger but less persistent innovations, a necessary feature to induce workers to increase their labor despite facing low wages. The estimates of the Drèze equilibrium imply a much higher wage rigidity. These features of the estimates under the Drèze equilibrium are somewhat inconsistent with reduced form VAR evidence and we do not consider this model an empirical success.

Further, because we also find that the approximations, both for the Drèze equilibria with Calvo pricing and for the union problem with Rotemberg pricing, are very close to the voluntary ex-post labor solutions, we are very confident that the latter series give us an accurate picture of the equilibria when the labor supply constraint is not violated.

Our conclusions are clear: the labor supply constraint in standard DSGE models with sticky wages can be frequently violated, and using demand-determined labor yields answers that are quite different from those that respect the labor supply constraint associated to agents not working more than what they want to.

[3]The labor supply constraint is binding occasionally, which requires the model to be solved using global method with a large number of state variables.

Though we document that the demand-determined allocation in the sticky wages environment is questionable, the Drèze equilibrium in the Calvo wage setting is not the only alternative to address this type of issues. One could, for instance, assume that when the demand-determined allocation is larger than the supply, wages could be reset (as in Hall and Milgrom (2008)). Another alternative is to introduce search and matching frictions that provide rents to workers with an employment relation, and to pose that under some circumstances the firm may ask them to work more or less hours than what they would choose on their own. Obviously, the Rotemberg wage setting with unions that take into account the labor supply constraint is yet another alternative. We believe that these alternatives are interesting and relevant, but no matter what alternative is taken, our results suggest that addressing the issue of avoiding violating the supply constraint may not have innocuous consequences.

In this paper, we focus on the Drèze equilibrium for Calvo settings because it has the appealing feature that it maintains the same primitive environment as the original model, and we can directly explore the logical implications of the Calvo wage setting in contrast to its demand-determined allocation. This exercise can be theoretically interesting by itself, and it extends the original Drèze equilibrium literature to a dynamic decision problem. The construction of the voluntary ex-post labor and the approximated Drèze equilibrium can be quantitatively relevant because they provide practitioners of DSGE models a simple tool to examine whether the laobr supply constraint is violated or not in their own models. For the Rotemberg wage setting we impose that the unions internalize the labor supply constraint which also maintains the physical environment of model.

Related Literature. — The central notion that we highlight in this paper is that agents should not work against their will, and that this implies a labor supply constraint that should be thought of as a participation constraint. Similar ideas have already been explored in the literature. Hall (2005) develops a search model with sticky wages to account for the observed labor fluctuations. The wage is reset only if it hits the boundary of the bargaining set which is between the minimum wage acceptable to the worker and the maximum wage acceptable to the employer. The workers' participation constraint has to be respected. In a similar fashion, Gertler, Sala and Trigari (2008) and Gertler and Trigari (2009) explore a search model with Calvo-style sticky wages, and whether the bargaining set is violated or not is checked ex-post. It is generally true that the bargaining set is large enough to accommodate Calvo-type sticky wages when agents are only subject to aggregate shocks, but it remains a question whether the bargaining set is large enough when agents also face idiosyncratic shocks. Recently, Christiano, Eichenbaum and Trabandt (2016) develop a quantitative model in which the wage is determined by alternating-offers bargaining, a variant of Hall and Milgrom (2008), a mechanism that introduces wage inertia endogenously, and is free of the concern on violating the participation constraint.

It can be argued along the lines of Barro (1977), that the form of wage rigidity in the Calvo model that the Drèze equilibrium maintains, is inconsistent with rational behavior,[4] and that the Calvo assumption should be changed. A possible alternative to the strict Calvo pricing rigidity assumption could be to specify additional circumstances under which prices or wages could change (violation of the labor supply constraint is one them). We take this possibility as a change of the physical environment and hence we choose not to pursue it in this paper. That being said, note that in Rotemberg models wages are reset every period, which is not subject to the Barro (1977) critique. As a result, the Calvo setting and the Rotemberg setting could be potentially viewed as two extreme benchmarks for other wage setting protocols when incorporating wage rigidities.

Our paper focuses on the willingness of agents to work, but a similar argument can also be made on the willingness of firms to produce goods at a fixed price. For example, Corsetti and Pesenti (2005) emphasize that firms should only produce if the ex-post price markup is larger than one. Bills (2004) and Alessandria, Kaboski and Midrigan (2010) consider firms' inventory stockout problems, where firms' sales have to be the minimum of the goods demanded and their existing inventory. Michaillat and Saez (2015) combine nominal rigidity with matching frictions in both goods and labor markets, where supply and demand jointly determine the outcome via affecting market tightness.

Van der Laan (1980), Kurz (1982), Dehez and Drèze (1984), Drèze (1997), and Citanna et al. (2001) are all related to Drèze (1975)'s original work, and study the properties of supply-constrained economy and explore the connection between price distortion and coordination failure. Herings (1996, 2014) extend Drèze (1975)'s work to settings with more flexible primitives and to dynamic environments. Bénassy (1993) compares the original Drèze equilibrium with other closely related disequilibrium concepts, and explore their implications in a static monetary economy with fixed prices and a fixed wage. Our paper differs from the previous literature in two ways. First, the market structure in our paper is monopolistic competition instead of perfect competition. Therefore, in periods where the wages can be reset, they will be set by forward-looking unions rather than the market.[5] Second, the previous literature focuses on equilibrium existence and multiplicity, while our paper explores the quantitative properties of Drèze equilibrium in a state-of-the-art DSGE model.

Organization. — We discuss the implicit assumption made in New Keynesian models when there is trade at non-market-clearing prices in Section I in the context of a model with wage setting à la Calvo. We proceed to explore in Section II the extent to which agents work against their will—what we jocularly label as

[4]Trujillo (1985), on the other hand, argued that rationality of conjectures can be defended and suffices to yield existence of equilibrium.

[5]Bénassy (1993) also considers the case where private agents set the prices and wages in a static environment, and in our paper agents need to solve a more complicated dynamic pricing problem.

slavery—in standard New Keynesian models (versions of Altig et al. (2011) and Smets and Wouters (2007)) and conclude that it happens too often to simply look the other way. Section III discusses what we think is the appropriate equilibrium concept for economies with wage settings à la Calvo, the Drèze equilibrium (Drèze, 1975), and compares its properties with those of the demand-determined allocation used in New Keynesian models and with those of an approximation to the Drèze equilibrium in various economies that we can solve. We then proceed to estimate a version of the Altig et al. (2011) model using the approximated Drèze equilibrium and we show that we obtain quite different estimates than those obtained when using demand-determined allocations in Section IV. Section V poses wage settings a la Rotemberg and explores the extent to which the labor supply constraint is violated. Section VI concludes by arguing that the approximation to the Drèze equilibrium should be used in lieu of the demand-determined equilibrium when studying environments with sticky wages.

I. The Labor Market in New Keynesian Models à la Calvo

We pose a typical New Keynesian model with sticky wages, first introduced by Erceg, Henderson and Levin (2000). There is a continuum of differentiated labor varieties n_i, $i \in [0, 1]$, which firms combine into a final labor input n for production using a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w:

$$n = \left[\int n_i^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} .$$

The wage w_i is set by unions that are specific to each labor variety i. Firms take all wages as given. Cost minimization, given wages and total labor n, yields demand schedules for each labor variety i,

$$n_i = \left(\frac{w_i}{w} \right)^{-\epsilon_w} n, \tag{1}$$

where w is an aggregate wage index $w = \left[\int w_i^{1-\epsilon_w} di \right]^{\frac{1}{1-\epsilon_w}}$ that satisfies $\int w_i \, n_i \, di = wn$.

A representative household consists of a continuum of workers, each one with different labor variety i that enjoys the same consumption level. The household's utility is given by

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i v(n_{i,t}) di \right) \right\} .$$

The union sets the wage to maximize agents' utility. The opportunity to reset the wage occurs with probability $1 - \theta_w$ (à la Calvo) every period. The union's problem is

$$\max_{w^*_{i,t}} \quad \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[u'(c_{t+k}) \frac{w^*_{i,t}}{p_{t+k}} n_{i,t+k} - v(n_{i,t+k}) \right] \right\}, \tag{2}$$

$$\text{subject to} \qquad n_{i,t+k} = \left(\frac{w^*_{i,t}}{w_{t+k}} \right)^{-\epsilon_w} n_{t+k}.$$

The first-order condition is

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[n_{i,t+k} u'(c_{t+k}) \left(\frac{w^*_{i,t}}{p_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} \frac{v'(n_{i,t+k})}{u'(c_{t+k})} \right) \right] \right\} = 0. \tag{3}$$

Although not stated explicitly, this problem assumes that firms can choose any quantity that they want of each labor variety, which requires that workers comply. Note that the worker is not choosing how much to work. If it did, it would choose ℓ_i to equate the real wage to the marginal rate of substitution (the standard intratemporal Euler condition):

$$\frac{w_{i,t}}{p_t} = \frac{v'(\ell_{i,t})}{u'(c_t)}. \tag{4}$$

We refer to the ℓ_i that solves equation (4) as the optimal labor supply under wage w_i.

In the absence of wage rigidity ($\theta_w = 0$), the union sets the wage every period and condition (3) becomes

$$\frac{w^*_{i,t}}{p_t} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{v'(n_{i,t})}{u'(c_t)}, \tag{5}$$

i.e. marginal revenue equals the marginal rate of substitution, or in standard parlance, the real wage is set to equal the marginal rate of substitution multiplied by the wage markup $\frac{\epsilon_w}{\epsilon_w - 1}$. Standard values for the elasticity of substitution ensure that what we call the labor supply constraint, $\ell_i \geq n_i$, that agents would like to work more than the quantity chosen by firms, is not violated and hence that the determination of the equilibrium quantity of labor via the quantity demanded is justified.

Under wage stickiness, however, the wage set by equation (3) may imply an optimal supply of labor $\ell_{i,t} < n_{i,t}$, violating the labor supply constraint. In this case, the assumption that labor is demand-determined implies that workers are working against their will (i.e., slavery).

What is the correct notion of equilibrium within the Calvo model in the context of a non-market-clearing price? Drèze (1975), following the disequilibrium models of Barro and Grossman (1971) and Malinvaud (1977), argued that it should be the minimum of supply and demand: trades should be voluntary. This is the notion that we follow in this paper.

Hours versus Bodies. — But is there anything really inappropriate about posing a model where agents work more than desired? Labor varies because of both changes in hours per worker and changes in the number of workers. An argument could be made that workers may not be free to choose the number of hours that they work without losing their jobs, and therefore our notion that workers should not work against their will only applies to the extensive margin. In that case, it is only when dealing with the extensive margin that the argument that the correct equilibrium condition is the minimum of the quantity supplied and the quantity demanded is really strong.

A recent wave of New Keynesian models (Galí (2011); Galí, Smets and Wouters (2012)) have incorporated unemployment by looking explicitly at changes in the extensive margin. In these models, households have a continuum of workers represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The i-dimension represents the type of labor service, while the j-dimension determines the worker's disutility from work, which equals j^γ if it is employed and zero if unemployed or outside the labor force. The household's utility is now given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \int_0^{n_{i,t}} j^\gamma \, dj \, di \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right).$$

An individual worker (i, j) takes the household's consumption level and the labor market conditions as given and will find it optimal to participate in the labor market if and only if

$$u'(c_t) \, \frac{w_{i,t}}{p_t} \geq j^\gamma. \tag{6}$$

Hence, the measure of workers in sector i who want to work is ℓ_i, which solves[6]

$$u'(c_t) \, \frac{w_{i,t}}{p_t} = \ell_{i,t}^\gamma. \tag{7}$$

We can (as Galí (2011) and Galí, Smets and Wouters (2012) do) define the unemployment rate as $u_t = \ell_t - n_t$. Moreover, in the absence of wage rigidities or in a steady state, the natural rate of unemployment rate u^n and the union's market

[6]Note that when the labor disutility function is $v(n) = \frac{n^{1+\gamma}}{1+\gamma}$, then equation (7) coincides with equation (4).

power are linked by

$$\frac{1}{\epsilon_w - 1} \approx \gamma u^n. \tag{8}$$

In these models, labor supply is determined by the number of agents willing to work. When labor demand exceeds labor supply (i.e.,s, $n_i > \ell_i$), some agents are required to work against their will (hence our jocular use of the term *slavery*). More dramatically, if labor demand exceeds the total population, $n_i > 1$, firms would be hiring workers that do not exist. It is in this type of model where the argument that the labor supply constraint should not be violated has the strongest appeal.

II. Are Agents Working Against Their Will?

We now turn to the quantitative exploration of the extent to which agents work against their will by comparing the properties of labor in our versions of the standard Altig et al. (2011) and Smets and Wouters (2007) environments with the level of labor in those economies that would be the minimum of supply and demand. Altig et al. (2011) augment Christiano, Eichenbaum and Evans (2005) with neutral and embodied technology shocks, while Smets and Wouters (2007) also include preferences shocks, wage and price markup shocks and government spending shocks. We use both of these models because they are de facto the standard New Keynesian models (more details about these models are in the online appendix). In these two models, labor is interpreted as hours worked and it could be argued that workers are implicitly obliged to work some periods more than they wish. For this reason, we also provide a discussion in Appendix A of the Galí, Smets and Wouters (2012) model where a unit of labor has the meaning of a worker. As we show there, the quantitative findings are very much in line with the findings of this Section.

To find out the extent to which agents are working against their will, we start solving and simulating the models as in the New Keynesian literature by assuming that labor is demand-determined. With the simulated history of aggregate wages and the aggregate labor demand, we construct the cross-sectional desired labor supply and labor demand for different labor varieties. We then construct our notion of voluntary ex-post labor by computing the minimum of the quantity of labor demanded and the quantity that agents would like to work for each wage/cohort and then adding them up across cohorts every period. The larger the difference between the demand-determined labor and the voluntary ex-post labor the more severe the violation of the labor supply constraint. Still, the voluntary ex-post labor is not the labor in a Drèze equilibrium because the latter requires that agents are aware of the equilibrium condition, and also of the implied adjustments in all the other model variables. However, this shortcut is useful in detecting whether we need to worry about this issue at all.

We discuss the details of how to construct the voluntary ex-post labor in Section II.A. This is not a trivial endeavor, because at any point in time, there are a large number of different wages, each one of them affecting a different group of workers who have different preferred labor choices. The quantitative analysis is in Section II.B.

A. *The Determination of the Voluntary Ex-post Aggregate Labor*

To determine the desired labor supply of workers we have to keep track, not only of the aggregate wage index of the economy, but also of the wages for all labor varieties i. Fortunately, this can be done by noting that all labor varieties that set the wage in a given period choose the same wage. We describe our procedure in three steps.

STEP 1: CONSTRUCT THE CROSS-SECTIONAL WAGE DISTRIBUTION. — The measure of workers that can reset their wages in the current period is $\mu_0 = 1 - \theta_w$, while the measure of workers with wage reset τ periods before is $\mu_\tau = (1-\theta_w)\theta_w^\tau, \quad \tau = 0, 1, 2, \ldots$, so μ_τ becomes negligible for τ large enough.

The simulation of the log-linearized model with demand-determined labor yields the sequence of the aggregate wage index $\{w_t\}$, which evolves according to

$$w_t = \left[\int w_{i,t}^{1-\epsilon_w}\, di\right]^{\frac{1}{1-\epsilon_w}} = \left[\theta_w (w_{t-1})^{1-\epsilon_w} + (1-\theta_w)(w_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}, \tag{9}$$

where w_t^* is the newly set wage in period t. Since we already have the aggregate wage sequence $\{w_k\}_{k=0}^t$, we can easily calculate the sequence of newly set wages $\{w_k^*\}_{k=0}^t$ using Equation (9). The wages prevailing in period t are then $\{w_{t-\tau}^*\}$, with corresponding measure $\mu_\tau, \tau \geq 0$.

STEP 2: CONSTRUCT CROSS-SECTIONAL LABOR DEMAND AND LABOR SUPPLY. — Given aggregate labor $\{n_t\}$, the labor demand for workers with wage rate $w_{t-\tau}^*$ is

$$n_{\tau,t} = \left(\frac{w_{t-\tau}^*}{w_t}\right)^{-\epsilon_w} n_t.$$

Agents that face wage rate $w_{t-\tau}^*$, have an optimal choice of labor given by the $\ell_{\tau,t}$ that solves

$$\frac{w_{t-\tau}^*}{p_t} = \frac{v'(\ell_{\tau,t})}{u'(c_t)}.$$

Aggregating both series over cohorts or wage groups, we obtain the aggregate demand for labor,[7] $n_t = \left[\sum_{\tau=0}^{\infty} \mu_\tau, n_{\tau,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$, and the aggregate supply of labor $\ell_t = \left[\sum_{\tau=0}^{\infty} \mu_\tau, \ell_{\tau,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$.

STEP 3: CONSTRUCT AGGREGATE LABOR. — Voluntary ex-post labor, e_t^p (we use the superscript p to denote that it is an ex-post quantity), is the minimum of supply and demand at each wage,[8]

$$e_t^p = \left[\sum_{\tau=0}^{\infty} \mu_\tau \left(\min\left\{n_{\tau,t}, \ell_{\tau,t}\right\}\right)^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{10}$$

We want to emphasize that e_t^p is not an equilibrium object, both because when making decisions, neither firms nor unions or workers take this factor into consideration, and because the implied path of consumption, investment, and capital is that associated with the demand-determined allocation. However, it allows us to check whether the extent to which the labor supply constraint is violated. If $n_{\tau,t} < \ell_{\tau,t}$ all the time, then $n_t = e_t^p$ and it is correct to use demand-determined labor. If instead, $n_{\tau,t} > \ell_{\tau,t}$ happens frequently and the difference between $n_{\tau,t}$ and $\ell_{\tau,t}$ is large, then n_t will be substantially different from e_t^p and the answers obtained by models that use demand-determined quantities of labor are questionable.

B. Quantitative Analysis of the Altig et al. (2011) and Smets and Wouters (2007) Models

Sticky wage models lack a straight identification of the steady-state wage markup, which affects the dynamics of labor and wages only through the slope of the wage Phillips curve that also depends on other deep parameters. The parameter ϵ_w that determines the steady-state markup is typically set exogenously. For example, Altig et al. (2011) sets the wage markup to be 5 percent, Smets and Wouters (2007) sets its value to 50 percent, and most DSGE models set this value between 5 percent to 25 percent.[9] Galí (2011) uses the relationship between the wage markup, the unemployment rate and the Frisch elasticity in (8), and explores

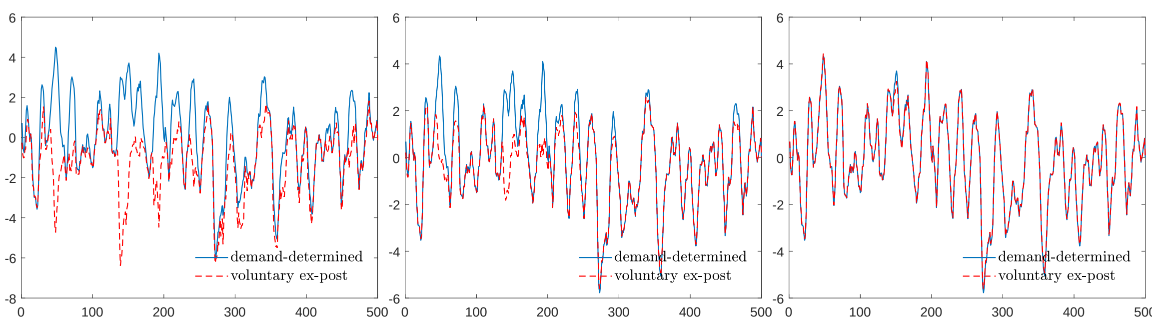
[7]Under log-linearization, an approximation error results in a negligible difference between aggregate labor and this expression.

[8]Quantitatively, the difference between the Dixit-Stiglitz aggregator and the linear average labor is negligible.

[9]Lewis (1986) surveys the literature on the wage premium for workers in a union, which corresponds to the wage markup in the model, and the value is between 10 percent to 20 percent. The steady-state wage markup is 5 percent in Christiano, Eichenbaum and Evans (2005), 15 percent in Chari, Kehoe and McGrattan (2002), and 20 percent in Levin et al. (2006).

markups from 5 percent to 25 percent for an empirically relevant range of Frisch elasticity.[10] In this paper we have chosen to estimate both of models, setting the wage markup to values in accordance with the recent literature, ranging from 5 percent to 25 percent. For values larger than 25 percent, the labor supply constraint turns out to be much less relevant.

The Labor Supply Constraint in the Altig et al. (2011) Model . — Figure 1 displays sample paths of the demand-determined labor (n_t) and of the voluntary ex-post labor e_t^p constructed as discussed in Section II.A for different wage markups. Note that demand-determined labor is always as large as the voluntary ex-post aggregate labor by construction. The difference between these two series is noticeable. Both series coincide in recessions, but the voluntary ex-post labor does not expand as much as the demand-determined labor in expansions. In fact, for a 5 percent wage markup, the voluntary ex-post labor actually declines when the demand-determined labor expands. This is because the expansion takes place by asking low-paid workers to supply a huge amount of labor which is no longer possible if the workers can choose not to meet the demand. Also, the smaller the wage markup, the larger the differences between these two series as the average distance between labor demand and labor supply shrinks.



5 percent wage markup 10 percent wage markup 25 percent wage markup

Figure 1. Sample Paths in Altig et al. (2011)

Table 1 summarizes the relevant statistics to compare both labor series for the Altig et al. (2011) economy for wage markups ranging form 5 percent to 25 percent. As discussed the violation of the labor supply constraint is more important the lower the wage markup. When the wage markup is 5 percent (the actual choice in Altig et al. (2011)), the voluntary ex-post labor is on average less than 1.41 percent lower than in the demand-determined. Almost 19 percent

[10]In an estimated version, Galí, Smets and Wouters (2012) obtain the steady-state wage markup with an value 18 percent.

Table 1—Labor Comparison in Altig et al. (2011)

	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
	5 percent wage markup				*10 percent wage markup*			
Demand-Determined	—	1.38	0.96	18.83	—	1.35	0.96	5.58
Voluntary Ex-Post	-1.41	0.97	0.42	—	-0.45	0.94	0.84	—
	15 percent wage markup				*25 percent wage markup*			
Demand-Determined	—	1.35	0.96	2.28	—	1.35	0.96	0.31
Voluntary Ex-Post	-0.15	1.18	0.93	—	0.00	1.33	0.96	—

Note: All the variables except the mean are logged and HP filtered except for the mean comparison. The column *labor violation* corresponds to the average measure of workers whose labor supply constraint is violated.

of labor is provided against the will of the workers. Perhaps, more importantly, the implied variance of the demand-determined labor series is 40 percent larger than that of the voluntary ex-post series and the correlation is more than twice as large. While for larger wage markups the differences are smaller, we find that even with a 15 percent wage markup the demand-determined labor series has a 14 percent larger variance than the voluntary ex-post labor series. For a 25 percent wage markup, the differences while positive are quantitatively negligible.

The Labor Supply Constraint in the Smets and Wouters (2007) Model . — Figure 2 displays sample paths of demand-determined labor and voluntary ex-post labor for the Smets and Wouters (2007) for a variety (5 percent, 10 percent and 25 percent) of wage markups. We see that the differences are very large, especially for low wage markups. The left panel of Table 2 displays the relevant statistics for those two series. The results are even more dramatic than for the Altig et al. (2011) economy: the graphs tell us that the differences are large and that they can still be clearly seen with a 25 percent markup. We see that the fraction of the labor force for whom the labor supply constraint is violated is as high as 45 percent with a 5 percent markup but even with 16 percent with a 25 percent markup. The differences in the mean labor are also very large ranging from almost 7 percent to almost 1 percent. The differences are really enormous for the variance of the two series, which now differ by a factor of 10 (for a 5 percent wage markup), and unlike for the Altig et al. (2011) economy it is larger for voluntary ex-post labor. The reason is that for a small wage markup, voluntary ex-post labor sometimes shrinks to the point of moving in the opposite direction, and hence what is an expansion under demand-determined labor is a recession in terms of voluntary ex-post labor (note the much lower correlation with output).

This reasoning also applies in accounting for the very low correlation between the voluntary ex-post labor and output. In this analysis, the labor supply constraint is severely violated in this economy, with the wage markup shock playing a central role in driving this result, so we look in more detail to the role played by this shock.

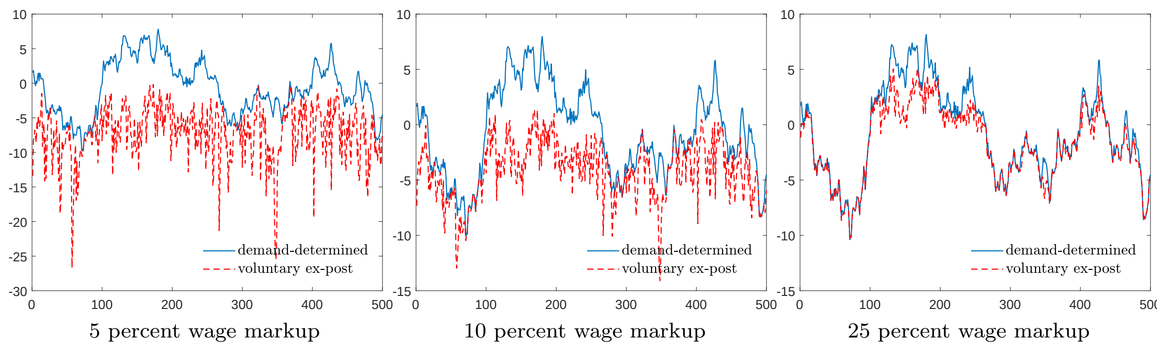


Figure 2. Sample Paths in Smets and Wouters (2007)

Wage Markup Shocks in the Smets and Wouters (2007) Economy. — The variance of the wage markup shock reported is 25.87 percent, quite a large value[11] that makes the implied wage markup itself sometimes close to zero or even become negative.

At a disaggregate level, the large volatility of markup shocks has some undesirable implications. Without log-linearization, the large variance of the shock implies implausibly large labor dispersion. Recall from eq. (1) that $n_{i,t} = \left(\frac{w_{i,t}}{w_t}\right)^{-\epsilon_{w,t}} n_t$. A negative wage markup implies that firms will be willing to demand more labor under a higher wage rate, which makes little economic sense. If the markup is positive but close to zero, then $\epsilon_{w,t}$ approaches to infinity, and as a result labor demand eq. (1) implies that almost all the labor is supplied by a tiny fraction of workers with the lowest wage, which is clearly counterfactual and the labor supply constraint is highly likely to be binding.[12] The log-linearized version of this model partially gets around this issue by making the demand of labor of type i independent of the actual realization of the markup shock: $\widehat{n}_{i,t} = -\epsilon_w \left(\widehat{w}_{i,t} - \widehat{w}_t\right) + \widehat{n}_t$, (where hats denote log-deviations from the steady state). However, the aggregate wage does depend on the aggregate wage markup leaving a channel through which

[11] That this is a huge value is also emphasized by Chari, Kehoe and McGrattan (2009).

[12] Moreover, for values of the wage markup close to zero in absolute value, the construction of the newly-set wage from eq. (9) may not even make economic sense as it may be a complex number.

Table 2—Smets and Wouters (2007) w/ and w/o Wage Markup Shock

	w/ Wage Markup Shock				w/o Wage Markup Shock			
	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
				5 percent wage markup				
Demand-determined	—	1.16	0.81	44.57	—	1.02	0.80	31.63
Voluntary ex-post	-6.73	11.76	0.01	—	-1.29	1.04	0.27	—
				10 percent wage markup				
Demand-determined	—	1.14	0.81	37.73	—	0.98	0.79	12.95
Voluntary ex-post	-3.53	3.24	0.16	—	-0.50	0.66	0.53	—
				15 percent wage markup				
Demand-determined	—	1.13	0.81	29.98	—	0.96	0.79	5.79
Voluntary ex-post	-2.14	1.73	0.32	—	-0.23	0.70	0.68	—
				25 percent wage markup				
Demand-determined	—	1.12	0.81	16.38	—	0.94	0.78	1.57
Voluntary ex-post	-0.84	1.07	0.59	—	-0.06	0.85	0.76	—

Note: All the variables except the mean are logged and HP filtered. The column *labor violation* corresponds to the average measure of workers whose labor supply constraint is violated.

this shock can generate by itself large violations of the labor supply constraint.[13] To address this concern, we also examine the labor supply constraint in the Smets and Wouters (2007) economy without the wage markup shocks. Figure 3 shows a sample path without markup shocks of the demand-determined labor and the voluntary ex-post labor series. We see immediately that the two series are much closer to each other, indicating that indeed the problem of violating the labor supply constraint may be much smaller without the labor markup shock.

Table 2 compares the demand-determined labor with the ex-post labor with and without wage markup shocks. Things are quite different, yet even without the wage markup shocks the labor supply constraint is violated quite often; there is also a sizeable reduction of average hours worked and a weakening of the correlation between labor and output; and last but not least, there are important differences in the variance of the labor series. For middle markups (10 percent, 15 percent) the variance of the demand-determined series is between 37 percent and

[13]See Chari, Kehoe and McGrattan (2009) for a discussion for different possible interpretations of the wage markup shocks.

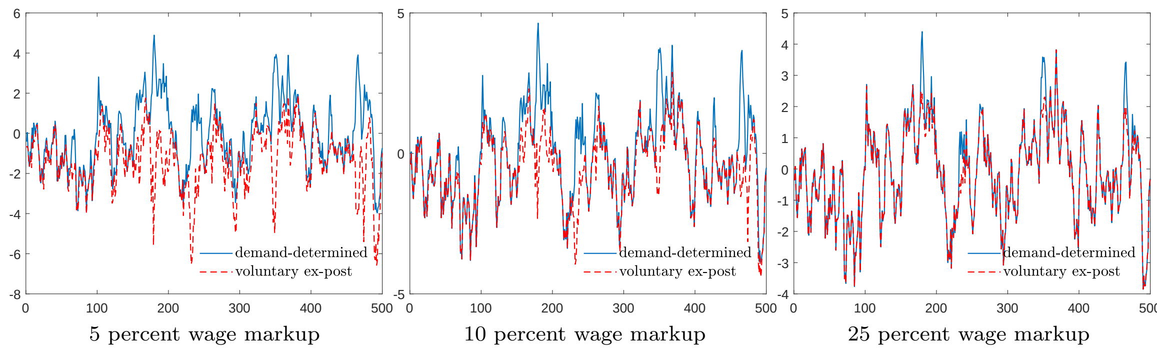


Figure 3. Sample Paths in Smets and Wouters (2007) w/o Wage Markup Shocks

48 percent larger than the voluntary ex-post series. Curiously, for the 5 percent markup version of the economy the opposite is true and the variance of the voluntary ex-post labor is slightly larger than that of the demand-determined series. The reason for this is, again, that for a small wage markup, voluntary ex-post labor sometimes shrinks to the point of moving in the opposite direction than the demand-determined, and hence what is an expansion under demand-determined labor may be a recession in terms of voluntary ex-post labor as the much lower correlation with output indicates.

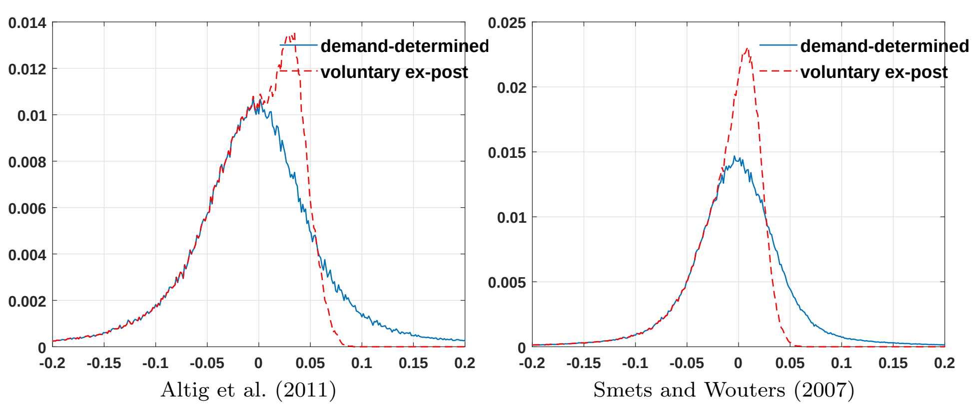


Figure 4. Cross-sectional Dispersion of Labor

Cross-sectional Dispersion of Labor across Sectors . — An implication of models with wage settings à la Calvo that is often ignored is that there is significant cross-sectional dispersion of hours across labor varieties i. In expansions, the increase in aggregate labor is mostly due to an increased labor demand for varieties that reset wages a while ago at a low level, and the varieties that reset wages more recently typically choose a high wage that can actually lead to a decline of the demand for their labor. The implied dispersion is especially large when the steady-state wage markup is relatively small. Figure 4 shows the cross-sectional distribution of hours for the Altig et al. (2011) and Smets and Wouters (2007) models for a 5 percent wage markup (without wage markup shocks). We see the large dispersion of the demand-determined labor and how the voluntary ex-post labor limits those varieties with high demand.

Table 3—Standard Deviation of Cross-Sectional Labor

	Altig et al. (2011)			Smets and Wouters (2007) without markup shocks		
Wage Markup	5 percent	15 percent	25 percent	5 percent	15 percent	25 percent
Demand-determined	0.069	0.039	0.029	0.080	0.046	0.035
Voluntary ex-post	0.046	0.035	0.029	0.048	0.038	0.033

Table 3 also illustrates this point for a variety of wage markups by showing the standard deviations of the cross-sectional demand-determined sector-specific labor demands and of the cross-sectional voluntary ex-post sector-specific labor demands. The dispersion is much larger for all demand-determined labor varieties than for the voluntary ex-post labor varieties.

III. Drèze Equilibrium

So far, we have made the case that the use of demand-determined labor as the equilibrium condition is inappropriate because households want to work less quite often: the minimum of the amount of labor demanded and supplied (as standard theory considers the appropriate equilibrium condition) behaves very differently than the amount of labor demanded. However, the series that we have constructed (voluntary ex-post labor) is not an equilibrium object because it is constructed along a path defined by the demand-determined labor and its associated series: output, consumption, investment, prices, wages, and so on. Moreover, the forecasts of agents are those of the demand-determined allocation. Therefore, we need to compute the Drèze equilibrium explicitly.

Unfortunately, log-linearization cannot be used to solve for the Drèze equilibrium. Global methods are needed given that the equilibrium condition is based on the min operator. Recent developments in computational economics that allow us to deal effectively with corner solutions, e.g. Guerrieri and Iacoviello (2015) cannot be applied either: these methods require the corners or temporarily binding constraints to be predetermined (like the zero bound of nominal interest rates or the lower and upper bound of hours worked), whereas in our economies, the min operator applies to two endogenous variables. We discuss this issue in detail in Appendix B. Moreover, the number of state variables is effectively infinite because the whole set of existing wages is part of the state vector (even if truncating the number of periods that we keep track of we would still need many state variables). Global methods can only be used with a limited number of variables, which presents a problem.

Our strategy here is to explore the properties of the Drèze equilibrium in an economy that we can solve with global methods (a simplified version of the Altig et al. (2011) economy with staggered wages à la Taylor), and to compare its solution with a suitable simple approximation (effectively one where we impose feasibility, and maintain properties of the log-linearized solution but forgo the rationality of agents' expectations). We claim that the global solution and our approximation are close, and hence we argue that we can use the approximated solution to the Drèze equilibrium as we do in Section IV.

We now describe the simple model with staggered wage contracts (Section III.A) and then describe an approximation to its solution that uses as a basis a log-linear approximation to the demand-determined equilibrium of the same economy (Section III.B). We compare the quantitative properties of both objects in Section III.C.

A. The Drèze Equilibrium

Consider an infinitely lived stochastic growth monetary economy. A representative household consists of a continuum of workers, each one with different labor variety i, that enjoy the same consumption level. The household's utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \frac{e_{i,t}^{1+\gamma}}{1+\gamma}\, di \right),$$

where $e_{i,t}$ is the labor of variety i. Households take prices and firms' profits as given, and their budget constraint is

$$p_t\Big(c_t + k_{t+1} - (1-\delta)k_t \Big) + \frac{1}{R_t} b_{t+1} = r_t^k k_t + \int_i w_{i,t}\, e_{i,t}\, di + b_t + \Pi_t.$$

Firms are competitive with Cobb-Douglas production technology $y_t = z_t k_t^{\alpha} e_t^{1-\alpha}$, where e_t is the final labor used in production aggregated via a Dixit-Stiglitz technology

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

and total factor productivity (TFP) follows an AR(1) process

$$\log z_t = \rho_z \log z_{t-1} + \zeta_t^z, \quad \zeta_t^z \sim \mathbb{N}(0, \sigma_z^2).$$

Unlike in standard New Keynesian models, labor market outcomes are determined by the minimum of supply and demand. We will introduce $\Phi_t(w_{i,t})$ and $\Psi_t(w_{i,t})$ below to denote the labor supply schedule and labor demand schedule as functions of wage rates. Individual firms and unions take them as given when making decisions. Because these functions also depend on the aggregate state, we use the time subscript to simplify notation, writing Ψ_t and Φ_t.

Firms are price takers and solve

$$\begin{aligned} \max_{k_t, e_t, e_{i,t}} p_t z_t k_t^\alpha e_t^{1-\alpha} - r_t^k k_t \quad &- \quad \int w_{i,t}\, e_{i,t}\, di \qquad \text{subject to} \\ e_t \quad &= \quad \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \\ e_{i,t} \quad &\leq \quad \Phi_t(w_{i,t}), \end{aligned} \tag{11}$$

where $\ell_{it} = \Phi_t(w_{i,t})$ is the maximum amount of labor the firm can obtain of labor of variety i under the wage rate $w_{i,t}$. In standard New Keynesian models, this last constraint is absent.[14] The solution to the firms' problems satisfies

$$\begin{aligned} \frac{r_t^k}{p_t} \quad &= \quad \alpha\, z_t\, k_t^{\alpha-1}\, e_t^{1-\alpha}, \\ e_{i,t} \quad &= \quad \min \left\{ \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^\alpha n_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t,\ \Phi_t(w_{i,t}) \right\}, \\ e_t \quad &= \quad \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \end{aligned}$$

Denote by $n_{it} = \Psi_t(w_{i,t})$ the desired labor demand in the absence of the quantity constraint. We have

$$\Psi_t(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^\alpha e_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t.$$

In this economy, there is a continuum of labor unions, each setting the wage of the type of labor that they represent, that maximize households' welfare given

[14] Constraint (11) implicitly assumes that all firms internalize that they are treated equally when facing a limited labor supply. We can see this as the result of assuming that firms send bids and that the available workers are equally distributed between all firms. This interpretation is strictly consistent with the model. An alternative that would add a lot of complexity without any substance is to pose a randomization mechanism.

the behavior of all other parts of the economy. Workers cannot be made to work against their will, and the union takes into account that there is an upper bound on the amount of labor that will be provided in their sector. The union chooses a nominal wage that will be effective for T^w periods:

$$\max_{w_t^*} \mathbb{E}_t \sum_{k=0}^{T^w-1} \left\{ \beta^k u'(c_{t+k}) \frac{w_t^*}{p_{t+k}} e_{i,t+k} - \frac{e_{i,t+k}^{1+\gamma}}{1+\gamma} \right\} \tag{12}$$

$$\text{subject to} \qquad e_{i,t+k} = \min \left\{ \left(\frac{u'(c_{t+k})}{\phi} \frac{w_t^*}{p_{t+k}} \right)^{\frac{1}{\gamma}}, \Psi_{t+k}(w_t^*) \right\}, \tag{13}$$

where $\Psi_{t+k}(\cdot)$ is the desired labor demand from the firm's side and the labor supply function $\Phi(w_{i,t})$ is given by

$$\Phi(w_{i,t}) = \left(\frac{u'(c_{t+k})}{\phi} \frac{w_{i,t}}{p_t} \right)^{\frac{1}{\gamma}}.$$

In the standard model, the constraint for the union is simply $e_{i,t+k} = \Psi_{t+k}(w_t^*)$.

In summary, we have defined three objects: labor supply in variety i, $\ell_{i,t} = \Phi_t(w_{i,t})$, labor demand in variety i, $n_{i,t} = \Psi_t(w_{i,t})$, and actual labor in variety i, $e_{i,t} = \min\{\ell_{i,t}, n_{i,t}\}$.

To complete the model, we include a simple Taylor type monetary policy rule:

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t,$$

where $\pi_t = \log \frac{p_t}{p_{t-1}}$ and y^* is the steady-state output level. The shock to the monetary policy rule follows an AR(1) process,

$$\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \mathbb{N}(0, \sigma_m^2).$$

The details of the numerical solution via global methods of this economy can be found in Appendix B.

B. *The Approximated Drèze Equilibrium*

Even in the simplified staggered wage model, computing the exact Drèze equilibrium is computationally intense. We therefore consider an approximation to the Drèze equilibrium which does not require the global solution. It has a much smaller computational burden and can be applied to medium-size DSGE models. As in our calculation of the voluntary ex-post aggregate labor, we also employ the log-linearized solution of the demand-determined allocation and then impose the ex post labor supply constraint. But unlike in the construction of the ex-post labor, the approximated Drèze equilibrium reconstructs all the main aggregate variables recursively, including capital, output, interest rate, and so on, guaranteeing that the resources constraints are satisfied. In what follows, we compare this approximation with the exact Drèze equilibrium, and we find that the allocations are very similar. As a result, we argue that the approximated Drèze equilibrium can be used to address questions in medium-size DSGE models where computing the exact Drèze equilibrium is extremely hard. Specifically, the construction of the approximated Drèze equilibrium consists of the following four logical steps.

Step 1: Log-linearize and solve the demand-determined equilibrium. — This is a standard step. The decision rules are required, not just a simulation.

Step 2: Recursively construct a voluntary ex-post measure of labor. — This step is what we described in Section II.A. The key difference is that there we use the sequence of capital stocks yielded by the demand-determined equilibrium, which may not be feasible. Thus, at this stage we construct a measure of the voluntary labor one period at a time, denoted as e_t^a. In this step we keep track of historical wages, w_t^a, which also include the information about the cross-sectional wage distribution.

Step 3: Recursively construct the main aggregate variables. — Here we use the labor in period t, e_t^a, and the previous period series of capital k_t^a to calculate output y_t^a (which is also used to construct the output gap). We then use the same policy function as in the demand-determined equilibrium to determine the newly set wage and price level. This is an approximation, since in the true Drèze economy, agents will take into account the possibility that the labor supply constraint may be binding. The interest rate R_t^a is set by using the reconstructed output gap. This part is mechanical.

Step 4: Determine consumption, investment, and next period capital. — This step is not mechanical. We have considered two possibilities: use the same consumption-to-output ratio or the same consumption of the demand-determined solution (investment is set residually to satisfy the resource constraint). We finally chose the same consumption because choosing the consumption-to-output ratio sometimes leads to countercyclical consumption. More specifically, in the demand-determined economy, after a positive technology shock, the consumption-output ratio is below its steady-state level because agents understand that it is better to increase investment to take advantage of the temporary high productivity. In the Drèze equilibrium, however, the response of labor is much more subdued with the same positive technology shock, which may lead to a much smaller expansion. If we used the low consumption-to-output ratio of the demand-determined allocation, there would be a recession rather than an expansion.

We do not want to argue that our approximation strategy is conceptually ideal, and we are aware that the allocation obtained in this approximation is subject to the fact that agents are not fully rational. The usefulness of this approximated equilibrium is simply justified by its small distance to the true Drèze equilibrium as we will show next.

C. A Comparison between the Drèze Equilibrium and Its Approximation

We now specify the staggered wage model quantitatively and solve for the Drèze equilibrium and for its approximation. The model has a large number of state variables to keep track of the wage distribution (see Appendix B for more details). The model period is a quarter and the annual interest rate in the steady state is 4 percent. The implied Frisch elasticity is 0.75 ($\frac{1}{\gamma}$), similar to estimates in Heathcote, Storesletten and Violante (2010). The labor share is 0.64, and the capital depreciation rate is 0.08 annually. The process for the TFP shock is similar to the one used in Ríos-Rull and Santaeulàlia-Llopis (2010). The monetary policy rule is the same as in Christiano, Eichenbaum and Rebelo (2011). The persistence of the monetary shock is 0.5, the same as in Galí (2008). We set the standard deviation of the innovation to the monetary shock to be 0.004. As discussed earlier, the most important parameter is ϵ_w, which determines the wage markup. The one we use here implies a 10 percent wage markup. If we apply the logic of equation (8), our choice of ϵ_w and γ leads to a 6 percent average unemployment rate[15]. We choose the duration of the wage contract to be four model periods, or one year.

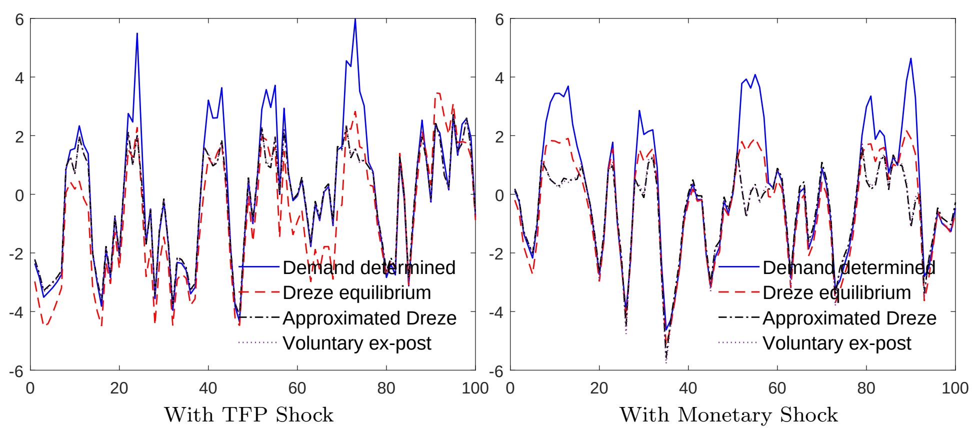


Figure 5. Sample Paths in the Staggered Wage Model

[15]Following Galí, Smets and Wouters (2012), the unemployment rate in sector i (the economy-wide counterpart is immediate) is $u_{i,t} = \log \ell_{i,t} - \log e_{i,t}$.

Figure 5 shows sample paths for the time series for labor for the four concepts that we are considering: the Drèze equilibrium, its approximation, voluntary ex-post labor and the demand-determined quantity of labor. We see here that for both shocks the Drèze equilibrium quantity of labor is similar to its approximation, and also to the voluntary ex-post quantity of labor and quite different than the demand-determined labor.

A similar picture arises from Table 4 that reports the properties of labor in the simple economy using one shock at a time. The Drèze quantity of labor, that of its approximation, and the voluntary ex-post labor series share similar volatility and cyclicality. The demand-determined allocation, however, is much more volatile than the others. More than 10 percent of agents work against their will in the demand-determined economy. To some extent surprisingly, the correlation with the Drèze equilibrium quantity of labor in the case of the monetary shocks is highest in the demand-determined solution.[16]

Table 4—Labor in the Staggered Wage Model

	mean	var	corr with output	corr with Drèze	labor violation
			TFP Shock		
Drèze Equil -0.33	1.60	1.00	1.00	—	
Approx Drèze Equil	-0.29	2.53	0.94	0.93	—
Voluntary ex-post	-0.32	2.60	0.89	0.93	—
Demand-Determined	—	3.80	0.96	0.96	10.42
			Monetary Policy Shock		
Drèze Equil	-0.33	1.60	1.00	1.00	—
Approx Drèze Equil	-0.40	1.27	1.00	0.78	—
Voluntary ex-post	-0.44	1.36	0.78	0.77	—
Demand-Determined	—	2.27	1.00	0.95	12.23

Note: All the variables except the mean are logged and HP filtered. The column *labor violation* corresponds to the average measure of workers whose labor supply constraint is violated.

Table 5 compares the business cycle properties of the main aggregate variables in the Drèze equilibrium, the approximated Drèze equilibrium, and the demand-determined economy (the voluntary ex-post solution only reconstructs the labor series, leaving other aggregate variables the same as the demand-determined solution). The volatility of variables in the Drèze equilibrium is similar to that in the

[16]This is due to the fact that with an expansionary monetary policy shock, the wage rate in the Drèze equilibrium is still set to accommodate the expansion, although to a lesser extent than the wage in the demand-determined solution. While the potential boom in the voluntary ex-post and the approximated Drèze equilibrium is often muted.

approximated Drèze equilibrium and the demand-determined economy is much more volatile than the other two.

Table 5—Business Cycle Statistics in the Staggered Wage Model

	TFP Shock			Monetary Policy Shock		
	Drèze Equil.	Approximated Drèze Equil.	Demand Determined	Drèze Equil.	Approximated Drèze Equil.	Demand Determined
		Variance			*Variance*	
Output	3.12	2.95	3.90	0.65	0.52	0.92
Labor	2.67	2.53	3.79	1.60	1.29	2.26
Consumption	0.17	0.17	0.19	0.01	0.01	0.02
Investment	42.82	38.10	52.11	10.12	7.96	13.95
		Correlation with output			*Correlation with output*	
Labor	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.61	0.57	0.62	0.61	0.57	0.62
Investment	1.00	0.99	1.00	1.00	0.99	1.00

Note: All the variables are logged and HP filtered.

We conclude that the approximation to the Drèze equilibrium built via log-linearization of the demand-determined solution and the recursive imposition of the minimum of the amount of labor supplied and demanded is a good approximation to a global solution of the Drèze equilibrium where the condition that labor is the minimum of the amount supplied and demanded is imposed ex-ante.

IV. Estimation of Altig et al. (2011) with the Drèze Equilibrium

So far, we have argued that in New Keynesian models with wage settings à la Calvo, the use of demand-determined labor yields allocations that are very different from those that the same parameterized model yields when labor is determined by the Drèze equilibrium where labor is the minimum of the amount supplied and the amount demanded. But this is not what really matters; perhaps different values of parameters yield similar properties between the two ways of determining the quantity of labor, and hence the answers that we obtain are the same. To settle this issue, we have to estimate the models under both types of labor determination.

The estimation of Smets and Wouters (2007) uses modern Bayesian methods that rely on the linearity of the model. Although demand-determined models are not linear, they are very well approximated by log-linear approximations and hence are extremely well suited for Bayesian or maximum likelihood estimation. The combination of the linearity and the Gaussian shock structure permits a relatively easy mapping from model parameters to its implied likelihood. The key feature of the Drèze equilibrium is its nonlinear nature, which unfortunately prevents us from applying standard linear Kalman filter techniques in evaluating the model's likelihood. The alternative nonlinear Kalman filter requires large computational power, which is only feasible for models with a relatively small number of state variables.

We can, however, estimate the approximated Drèze equilibrium in Altig et al. (2011), the other central model in the New Keynesian literature. Altig et al. (2011) and its precedent Christiano, Eichenbaum and Evans (2005) estimate a medium-scale DSGE model by matching the impulse responses of various variables to different shocks. The impulse responses are recovered from the estimation of a certain structural vector autoregression (VAR) model. The identification strategy in Altig et al. (2011) is similar to that in Christiano, Eichenbaum and Evans (2005), where only nominal variables like the velocity of cash balances respond to contemporaneous monetary policy shocks but not the real variables such as hours, consumption, investment, and so on. It is also assumed that monetary policy is set conditional on the current values of real variables and only on the past values of nominal variables. In addition, innovations to technology (both neutral and capital embodied) are the only shocks that affect long-run labor productivity, and capital embodied technology shocks are the only shocks that affect the long-run relative price of investment goods. Crucial to this endeavour is the ability to identify the shocks, something that can be done with the three shocks in Altig et al. (2011).

The parameters of the model are chosen in such a way that the model's impulse responses to the structural shocks match their counterpart estimated from the data. In particular, three structural shocks are considered: a monetary shock, a neutral technology shock, and an embodied investment technology shock. The estimation method is generalized method of moments (GMM), which only requires the impulse response of the model.[17] Because the likelihood of the model is not required, we can apply this estimation method to the approximated Drèze equilibrium using the same exogenously calibrated parameters than Altig et al. (2011).

Table 6 shows the properties of the estimates of the approximated Drèze equilibrium and of the demand-determined allocation in the Altig et al. (2011) model with a 5 percent markup. Our interpretation of these very different sets of estimates is that the unwillingness of households in the Drèze equilibrium to work a lot under some circumstances requires that other pieces of the model have to do a lot more work to create the observed fluctuations:

1) The neutral technology shock is dramatically affected. To induce more movement in labor, the estimated shock is now both much more volatile and less persistent: the unconditional variance of the neutral technology shock is 0.039 in the Drèze equilibrium relative to 0.024 in the demand-determined allocation. A larger, but less persistent, shock makes households

[17] The weighting matrix of GMM is diagonal with the inverse of the standard deviations of the impulse responses estimated in the structural VAR.

Table 6—Estimated Parameter Values

	Demand-Determined	Approximated Drèze
Std Dev of neutral tech shock, $\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)
Autocor neutral tech shock, ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)
Std Dev of monetary shock, σ_M	0.331 (0.084)	0.325 (0.078)
Autocor monetary policy shock, ρ_M	-0.037 (0.111)	-0.040 (0.130)
Std Dev of embodied tech shock, $\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)
Autocor embodied tech shock, ρ_{μ_Υ}	0.241 (0.224)	0.318 (0.176)
Wage rigidity, ξ_w	0.722 (0.123)	0.825 (0.043)
Price rigidity, γ	0.040 (0.029)	0.054 (0.039)
Variable capital utilization, σ_a	1.995 (2.222)	4.564 (7.070)
Investment adjustment cost, S''	3.281 (2.038)	4.752 (2.378)
Interest elasticity of money demand, ϵ	0.808 (0.208)	0.779 (0.193)
Habit formation, b	0.706 (0.045)	0.698 (0.058)
Effects of neutral tech shock on policy, ρ_{xz}	0.343 (0.266)	0.195 (0.480)
Effects of embodied tech shock on policy, $\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)
Scaling factor of neutral tech shock, c_z	2.997 (2.310)	1.027 (0.749)
Scaling factor of neutral tech shock, c_z^p	1.327 (1.381)	0.665 (0.650)
Scaling factor of embodied tech shock, c_Υ^p	0.135 (0.244)	0.107 (0.268)
Scaling factor of embodied tech shock, c_Υ	0.246 (0.244)	0.305 (0.266)

Note: The estimation is with 5 percent wage markup. The magnitude of shocks that generate the impulse response functions are set to their standard deviations.

more willing to supply labor.

2) The rigidity of wages and prices is somewhat larger. The lower response of labor in the Drèze equilibrium also requires, perhaps a bit counterintuitively, larger rigidities in the model to generate more fluctuations. This is true both for wages, where the Drèze equilibrium is imposed, and for prices, where it is not.

3) Two other pieces of the model are now larger. The role of variable capital utilization is now value, as is investment adjustment cost parameter. Still, these two parameters are somewhat imprecisely estimated and we should not insist on them.

Table 7—Labor Comparison with Different Estimation Strategies

	Estimated with Demand-Determined				Estimated with Approximated Drèze			
	mean	var	corr w/ output	labor violation	mean	var	corr / output	labor violation
				Neutral Technology Shock				
Demand-Determined	—	0.18	0.87	15.09	—	0.24	0.97	19.03
Approximated Drèze	-1.57	1.16	0.96	—	-2.59	1.41	0.95	—
				Investment Technology Shock				
Demand-Determined	–	0.67	0.99	6.22	–	0.52	0.99	7.89
Approximated Drèze	-0.42	0.34	0.98	—	-0.55	0.32	0.99	—
				Monetary Shock				
Demand-Determined	—	0.46	1.00	2.56	—	0.33	1.00	1.15
Approximated Drèze	-0.07	0.33	0.99	—	-0.01	0.30	1.00	—
				All Shocks				
Demand-Determined	—	1.38	0.96	18.83	—	1.15	0.95	22.63
Approximated Drèze	-2.28	2.06	0.98	—	-3.41	1.99	0.96	—

Note: Numbers are in percentages except for the correlation with output.

Table 7 shows what the different solutions yield for each of set of estimates obtained. The left panel of the table shows the effects of the processes estimated via the demand-determined solution for labor when we look both at the demand-determined solution and at the approximated Drèze equilibrium. The right panel shows the effects of the processes estimated with the approximated Drèze equilibrium when we both look at the demand-determined solution and at the approximated Drèze equilibrium. The numbers in boldface are the properties of the economies when they are used to estimate the parameters. The Table shows some other important features of the differences between the demand-determined solution and the approximated Drèze equilibrium:

4) The estimates of the approximated Drèze equilibrium increase the role of the neutral technology shock. The variance of labor is larger using both

equilibrium notions relative to the demand-determined estimates. Comparing the original Altig et al. (2011) results with the Drèze equilibrium under the new estimates, the contribution of the neutral technology shock to the variance of labor increases from 13 percent to 71 percent, that of the investment or embodied technology shock shrinks from 49 percent to 16 percent, and that of the monetary shocks also shrinks from 33 percent to 15 percent.[18]

5) Under both sets of estimates, the variance of labor is much larger in the approximated Drèze equilibrium. The unwillingness of households to work under many circumstances generates recessions that are not present in the demand-determined solution.

There are two main takeaways from this exercise. First, addressing the violation of the labor supply constraint does not have innocuous consequences for the model. The estimates under the Drèze equilibrium are significantly different than those obtained under the demand-determined solution. Second, the particular approach we used to address the issue, the Drèze equilibrium, does not improve the empirical performance of DSGE models with sticky wages. To induce workers to increase their labor supply, the required persistence for TFP process is much lower and the required innovation is much more volatile that what most economists think is the case. Also, the fraction of fluctuations that are accounted by TFP shocks becomes much higher. The model with Drèze equilibrium brings the results further away from the more reduced-form evidence. Even though the Drèze equilibrium is the natural candidate in sticky wage models to avoid the violation of the labor supply constraint in models with Calvo wage setting, perhaps other alternatives should be explored to achieve the goal that it can simultaneously respect agents' willingness to work and fit the aggregate time series. Such alternatives are likely going to depart from the strict Calvo environment.

Robustness of findings. — The estimation method of Altig et al. (2011) is to minimize the distance between the impulse response of an structural VAR in the data and the theoretical VAR of the model, a procedure that works cleanly with a model that is linear (or where its log-linear approximation is deemed to be accurate enough). However, the Drèze equilibrium is very non linear which raises two concerns. First, the impulse response function depends on the size of the size shock that is used to evaluate it. Figure 6 illustrates this point. The impulse responses of labor to a neutral technology shock are plotted for the data, and for the demand-determined solution and the Drèze equilibrium approximation for a 10 percent markup for different size shocks (1, 2.5 and 3 times the standard

[18] As the quick-witted reader may have noticed, the contributions of the orthogonal shocks to the variance of labor add up to slightly above 100 percent. The reason for this is the nonlinear nature of the model. Fortunately for our analysis, the differences are quite small, and the contribution of each individual shock when the others are shut out gives a good picture of their overall contribution.

deviation). We see how both the data and the demand-determined solution scale nicely as the shock gets larger, but not the Drèze equilibrium where the expansion turns into a recession within a couple of years. The second concern arises as a result of the first one and can also be seen in the picture: the demand-determined and the Drèze impulse response are equal for small values of the shocks and different for large values of the shock. This is likely to be generally the case, as for small values of the shock the labor supply constraint is unlikely to be binding. Consequently, impulse response functions of the Drèze equilibrium to small shocks hide the violations of the labor supply constraint.

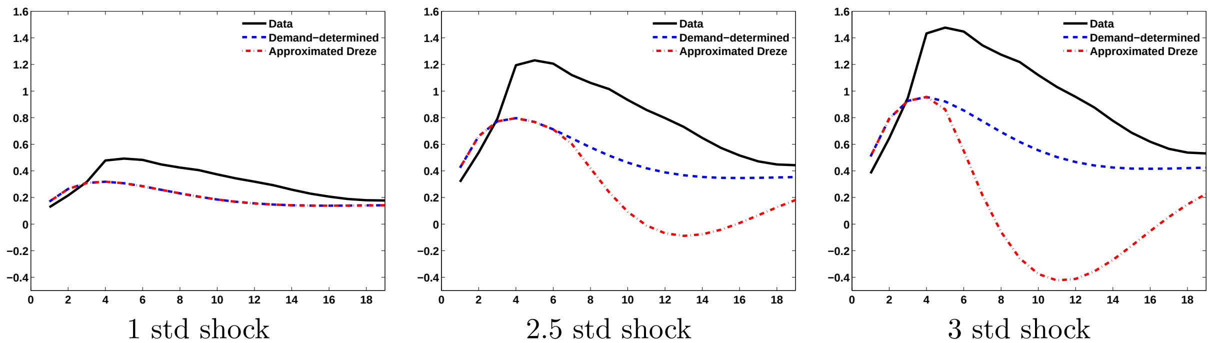


Figure 6. IRF of Labor to Neutral Tech. Shock

Note: The parameters to generate the impulse responses are estimated under demand-determined solution with 10 percent wage markup.

To deal with this issue, we have reestimated the Drèze equilibrium with the impulse response of a shock of size 1.5 standard deviations and we report them in the online appendix. The findings that we have reported are replicated in this case, if anything the estimate of the autocorrelation of neutral productivity shocks becomes even smaller than before. We also report in the online appendix the estimates that we obtain for 10 percent and for 15 percent markups obtained targeting the impulse response to various sizes of the shocks (larger than one standard deviation for the reasons adduced before). The findings are again confirmed, the neutral technical shock is much more important than in the demand determined economy, with the differences shrinking as we look at economies with a larger markup. We conclude the exploration of this issue by estimating both the demand-determined and the Drèze economies for larger markups using impulse responses to larger shocks. Again the same features reappear, the role of neutral technical change increases albeit with a much larger variance in the Drèze equilibrium relative to the demand-determined solution. We conclude that using larger markups and larger shocks to specify which impulse responses the model attempts to replicate generates patterns similar to those in our baseline specification of the Altig et al. (2011) model.

V. Wage Adjustment Costs à la Rotemberg

A popular alternative to nominal wage rigidity à la Calvo is the wage-adjustment cost mechanism proposed by Rotemberg (1982), where the nominal wage can be reset every period, but there is a quadratic adjustment cost when its value is changed. As shown in Born and Pfeifer (2016), by suitably choosing the adjustment quadratic cost parameter, the aggregate wage dynamics in the Rotemberg model and the Calvo model are identical up to a first order approximation. But being observationally equivalent at the aggregate level does not imply that the cross-sectional labor allocation and the extent to which the labor supply constraint is violated is the same: in the Calvo model, the aggregate wage is less volatile than the wages for each labor variety, while in the Rotemberg model, a common wage prevails in the economy and is reset every period. In the Calvo model there are workers whose wage was set at a low level long time ago and hence they are likely to have their labor supply constraint violated making it natural to expect that the labor supply constraint will be binding more frequently than in the Rottemberg model.

Meanwhile, as mentioned in the introduction, the Calvo pricing assumption may be too rigid and one may want to specify additional circumstances under which prices or wages could change (violation of the labor supply constraint being one of them). The Rotemberg model could be viewed as an example where wage rigidity is introduced without the strong assumption that wages cannot be changed no matter what. As a result, it is not subject to the Barro (1977) critique.

To explore the extent to which violating the labor supply constraint is quantitatively relevant in the Rotemberg model, we revisit the Smets and Wouters (2007) and Altig et al. (2011) models under a Rottemberg wage adjustment mechanism and we compare its performance relative to the restriction that the amount of labor cannot exceed what workers are willing to work.

Consider economies like the ones described in the previous sections except in the fact that labor unions can change the wage every period subject to a quadratic wage adjustment cost. The variety i union's problem differs from that in Equation (1) and becomes

$$\max_{\{w_{i,t+k}\}} \quad \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[u'(c_{t+k}) \left(\frac{w_{i,t+k}}{p_{t+k}} n_{i,t+k} - \frac{\vartheta}{2} \left(\frac{w_{i,t+k}}{w_{i,t+k-1}} - 1 \right)^2 y_{t+k} \right) - v(n_{i,t+k}) \right],$$

$$\text{subject to} \qquad n_{i,t+k} = \left(\frac{w_{i,t+k}}{w_{t+k}} \right)^{-\epsilon_w} n_{t+k},$$

where parameter ϑ determines the size of the wage adjustment cost which is assumed to be proportional to nominal output $p_t y_t$. Here, the union implicitly assumes that workers are always willing to supply amount $n_{i,t+k}$ of labor. Due to the equivalence of the Rotemberg model and the Calvo model after linearization, we choose a Rotemberg cost parameter ϑ so that it will imply the same wage

Phillips curve as that in the Calvo model.

To see whether the standard characterization of Rottemberg pricing (the demand-determined solution) violates the requirement that agents do not work more than what they want we compare the demand-determined quantity of labor that solved the previous problem, n_t^*, with the minimum of this demand-determined quantity and the actual amount of work that agents are willing to supply, this is, the ℓ_t^* that solves

$$\frac{w_t}{p_t} = \frac{v'(\ell_t)}{u'(c_t)}.$$

We denote this $\min\{n_t^*, \ell_t^*\}$ the ex-post labor. Note that in this case we do not need to construct the cross-sectional labor demand and labor supply as we did in Section II.A for the Calvo type economies since all labor types have the same wage in equilibrium.

Table 8—Altig et al. (2011) with Rotemberg Adjustment Costs

	5 percent wage markup				10 percent wage markup			
	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
Demand-Determined	—	1.38	0.96	5.37	—	1.35	0.96	0.06
Voluntary Ex-post	-0.07	1.24	0.93	—	0.00	1.35	0.96	—

Note: All the variables except the mean are logged and HP filtered. The column *labor violation* corresponds to the frequency of the labor supply constraint violation. The wage markup used is 5 percent.

Tables 8 and 9 compare the properties of the quantities of labor allocations implied by ignoring workers willingness to work with those that arise when such constrained is taking into account. The labor supply constraint is still sometimes violated with Rottemberg adjustment costs, although as expected, less often than in economies with Calvo pricing. In the Altig et al. (2011) economy, the labor supply constraint binds 5 percent of the time, and labor volatility is 11 percent larger when ignoring the labor supply constraint than when imposing it with a 5 percent markup, while it is essentially identical with a 10 percent markup.

In the Smets and Wouters (2007) without wage markup shocks, and with a 5 percent markup the labor supply constraint binds 19 percent of the time and has 23 percent larger labor volatility when we ignore the labor supply constraint. There are almost no differences with a larger markup. When the wage markup shocks are also included, the differences between the economy that ignores the labor supply constraint and that where we prevent that constraint to be violated are much more dramatic. For a 5 percent markup the constraint is violated 36 percent of the time, leading to a labor volatility 16 percent larger and a correlation

Table 9—Smets and Wouters (2007) with Rotemberg Adjustment Costs

	w/o wage markup shock				with wage markup shock			
	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
				5 percent wage markup				
Demand-Determined	—	1.01	0.80	18.48	—	1.16	0.82	35.79
Voluntary Ex-post	-0.19	0.82	0.69	—	-1.12	1.00	0.49	—
				15 percent wage markup				
Demand-Determined	—	0.95	0.79	0.19	—	1.13	0.81	15.55
Voluntary Ex-post	0.00	0.95	0.79	—	-0.41	1.07	0.68	—
				25 percent wage markup				
Demand-Determined	—	0.93	0.78	0.00	—	1.12	0.81	5.50
Voluntary Ex-post	0.00	0.93	0.78	—	-0.12	1.09	0.77	—

Note: All the variables except the mean are logged and HP filtered. The column *labor violation* corresponds to the frequency of the labor supply constraint violation.

with output also much larger in the unconstrained economy. With a 15 percent markup the constraint binds 16 percent of the time, volatility is still 5 percent larger and labor is clearly more correlated with output. Even with a the 25 percent markup the differences are noticeable: hours are 3 percent more volatile and the correlation is .04 larger than in the constrained economy.

To summarize, with a Rotemberg wage setting mechanism the issue of agents working against their will is also present. In the Altig et al. (2011) economy this happens only when the markup is no higher than 5 percent. In the Smets and Wouters (2007) economy this happens also with markups no higher than 5 percent when we ignore wage markup shocks. But when we include them, the labor supply constraint becomes very relevant as ignoring it yields noticeably larger labor volatility and correlation between labor and output than when it is imposed.

However, like in the Calvo economies, the ex-post imposition of the labor supply constraint does not give us a complete picture of what is the behavior of the economy when the quantity of labor violates the labor supply constraint. We should explicitly incorporate the constraint as part of the equilibrium. Because the unions reset prices each period, the imposition of the labor supply in the wage setting problem is more straightforward and the explicit implementation of the equilibrium as a Drèze equilibrium is no longer necessary. In particular, we add the following constraint to the union's problem

$$n_{it} \leq \ell_{it}, \qquad \text{where } \ell_{it} \text{ solves} \qquad \frac{w_{it}}{p_t} = \frac{v'(\ell_{it})}{u'(c_t)}. \tag{14}$$

Note that the unions' problem is one with occasionally binding constraints where the actual constraint limit is a varying one which precludes the use of linear methods. We cannot compute the original Altig et al. (2011) or Smets and Wouters (2007) models with the occasionally binding constraint, but we are able to solve a simpler economy explicitly with unions that internalize the labor supply with the use of global methods. This simple economy is similar to the one considered in Section III.A. The differences are: (1) we replace the Taylor staggered wage contract with the Rotemberg wage setting; (2) the firms are not subject to the quantity constraint (11). We set ϑ such that the implied Calvo parameter $\theta_w = 0.75$. For this simple economy, we can also define an approximate solution exactly like we did in Section III.C and see how it compares with the exact solution and with the solution with an ex-post implementation of the labor supply constraint (for comparison we also look at the allocation that ignores the constraint).

Table 10—Labor in the Rotemberg Model for Various Solutions

	TFP Shock				Monetary Policy Shock			
	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
Ex-ante Constrained	-0.39	3.95	0.99	—	-0.76	2.63	1.00	—
Approx. Ex-ante	-0.11	3.93	0.99	—	-0.28	2.78	1.00	—
Voluntary Ex-post	-0.14	3.92	0.97	—	-0.38	2.79	0.91	—
Demand-Determined	—	4.31	0.99	6.78	—	3.42	1.00	12.06

Note: All the variables except the mean are logged and HP filtered. The column *labor violation* corresponds to the frequency of the labor supply constraint violation.

Table 10 reports the properties of the various labor allocations. We see that the ex-ante labor constraint is very similar to the approximated ex-ante and also to the economy with the ex-post constraint (slightly less so for the monetary policy shock) and they are all very different than the economy that ignores the labor constraint. We conclude that the use of the ex-post labor allocation for the Altig et al. (2011) and the Smets and Wouters (2007) economies gives as a reasonable picture of how the equilibrium that takes into account the labor supply constraint would look.

VI. Conclusion

In this paper, we have explored what happens in the canonical New Keynesian models when the demand-determined solution for labor is replaced by a solution that ensures that the quantity of labor used in the economy is not larger than the quantity of labor that agents are willing to supply. In economies with wage setting à la Calvo this is accomplished by using the Drèze equilibrium (or an approximation to it), while in economies with wage adjustment costs à la Rotemberg this is accomplished by imposing the non-violation of the labor supply constraint (or an approximation to it) to the wage setting unions.

We have argued that the differences are large. Typically, between 5 percent and 30 percent of the labor force is working against agents' will on average in a demand-determined solution depending on the level of the wage markup. Comparing the demand-determined solution with equilibrium allocations that satisfy the labor supply constraint in standard models, we see substantially different labor volatilities, usually (but not always) larger in the demand-determined models. The problem is somewhat less dramatic in Rotemberg style settings but still yields quite different properties even for large wage markups when wage markup shocks are taken into account.

More importantly, perhaps, when we estimate the Drèze equilibrium in the economies with wage setting à la Calvo, it yields answers that are substantially different from those provided by the demand-determined solution estimates: in the context of the Christiano, Eichenbaum and Evans (2005) and Altig et al. (2011) economy, the role of neutral technology shocks rises from 13 percent to 70 percent, these shocks become larger and less persistent, and the estimates of the rigidities become larger.

These findings are dependent on the particular wage markup of the economy. With wage markups sufficiently large, the problem becomes almost (but not completely) nonexistent. Still our calculations are made for what we think are the most empirically relevant values of the markup.

We conclude by encouraging researchers to be more concerned about the problem that wage rigidity causes in worker's willingness to work, and to either incorporate it the labor supply constraint explicitly in their models or to consider models of wage rigidity less prone to this problem, perhaps a version of the Calvo model where wages could also be changed when the labor supply constraint binds.

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Appendix

Analysis of Galí, Smets and Wouters (2012)

Galí, Smets and Wouters (2012) set the steady-state wage markup level to 18 percent, which via equation (8) implies a Frisch elasticity of 0.25. The approach in Galí, Smets and Wouters (2012) allows to separately identify labor supply shocks and wage markup shocks, and the estimated standard deviation of the wage markup shock is much smaller than that of Smets and Wouters (2007) (0.04 versus 0.25).[19]. Table A1 shows the comparison between demand-determined labor and the voluntary ex-post labor. The pattern is broadly consistent with the findings in Section II.B. The main difference is the role played by the wage markup shocks become smaller, and the violation of the labor supply constraint becomes less severe, even though the two series are still significantly different from each other.

Although the Galí, Smets and Wouters (2012) estimate of the standard deviation of the wage markup shock is much smaller than that of Smets and Wouters (2007) (0.04 versus 0.25). Table A1 shows the comparison between demand-determined labor and the voluntary ex-post labor. The pattern is broadly consistent with the findings in Section II.B. The main difference is the role played by the wage markup shocks become smaller, and the violation of the labor supply constraint becomes less severe, even though the two series are still significantly different from each other.

Table A1—Labor Comparison in Galí, Smets and Wouters (2012)

	with Wage Markup Shock				w/o Wage Markup Shock			
	mean	var	corr w/ output	labor violation	mean	var	corr w/ output	labor violation
Demand-Determined	—	0.54	0.76	16.60	—	0.51	0.75	3.89
Voluntary Ex-Post	-0.57	0.56	0.59	—	-0.13	0.39	0.69	—

Note: All the variables except for the mean of labor are logged and HP filtered.

[19]In the simulation, it still generates negative wage markup from time to time.

Details of the Computation of the Staggered Wage Economy

We use a policy function iteration method to obtain the numerical solution. The system of equations that characterizes the solution is

$$
\begin{aligned}
c_t^{-\sigma} &= \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right], \\
c_t^{-\sigma} &= \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1 + r_{t+1}^k - \delta}{\pi_{t+1}} \right], \\
\frac{r_t^k}{p_t} &= \alpha \; z_t \; k_t^{\alpha-1} \; e_t^{1-\alpha}, \\
\log R_t &= \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t, \\
c_t + k_{t+1} &= y_t + (1-\delta) k_t, \\
y_t &= z_t \; k_t^{\alpha} \; e_t^{1-\alpha}, \\
e_t &= \left[\sum_{i=0}^{T_w} e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \\
e_{i,t} &= \min \left\{ \left[\frac{w_{t-i}^*}{(1-\alpha) z_t k_t^{\alpha} e_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t, \; \left(\frac{u'(c_t) w_{t-i}^*}{\phi p_t} \right)^{\frac{1}{\gamma}} \right\}. \qquad \text{(B1)}
\end{aligned}
$$

In addition, the optimization problem (12) to (13) is also part of the system.

There are two differences between the demand-determined economy and the Drèze equilibrium: first, the employment is determined by the minimum of the demand and supply in the Drèze equilibrium (see equation (B1)), whereas in the demand-determined economy, employment always equals to labor demand. Second, the choice of the optimal nominal wage, w_t^*, cannot be characterized by a simple first-order condition because of the potential binding labor supply constraint. In the computation, we have to use a global search method to find the optimal wage choice.

We look for policy functions for $\{k_{t+1}, c_t, y_t, e_{i,t}, e_t, w_t^*, \pi_t, R_t\}$. The state variables at period t include the following: the current technology shock z_t or the monetary shock η_t, the capital stock k_t, and the wages set in the previous three periods $\left\{ \frac{w_{t-1}^*}{p_{t-1}}, \frac{w_{t-2}^*}{p_{t-1}}, \frac{w_{t-3}^*}{p_{t-1}} \right\}$.

Occasionally Binding Constraint. — Guerrieri and Iacoviello (2015) develop a Dynare toolkit that can solve DSGE models with occasionally binding constraints. In this subsection, we discuss why this method cannot be applied in our model.

Guerrieri and Iacoviello (2015)'s method can handle problems with exogenous binding constraints such as non-negative investment, a zero-bound on the nominal interest rate, an exogenous borrowing constraint, and so on. In these cases, one can neatly partition the problem into two regions: in the first region, the constraint is not binding and one can use the first-order condition to characterize the solution. In the second region, the constraint is binding and one can simply set the variable to equal the constraint (for example, let the nominal interest rate be zero).

The problem in this paper is more involved. When the labor supply constraint is not binding, employment equals the labor demand, and the first-order condition can be applied as in the standard New Keynesian literature. When the labor supply constraint is binding, different from the examples listed earlier, the labor supply constraint is not an exogenous constraint because the level of the labor supply is endogenously determined. What makes this case even worse is that when the labor supply constraint is binding, the union's problem is not concave, which implies that we cannot use either the first-order condition or some exogenous value to determine the optimal wage (and hence employment). Therefore, we solve the Drèze equilibrium using a global method.