Contagious Bubbles*

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August, 2023

Abstract

This paper proposes a framework to study contagious stock price bubbles in a multi-industry economy with heterogeneous firms. Rational stock price bubbles arise endogenously under financial frictions to improve firms' equity positions and liquidation values. We characterize the full set of bubble equilibria and provide conditions under which the equity values of firms in different industries contain each other's bubbles, such that a bubble burst in a critical industry can cause bubbles to burst in other industries, but not necessarily vice versa. We calibrate the financial linkages in our model using U.S. merger and acquisition data and show quantitatively that the financial industry is critical for the existence of contagious bubbles.

Keywords: Financial Networks, Asset Bubbles, Financial Contagion.

JEL Codes: E2; E44; G1

*We are grateful to Wei Cui for discussing our paper, and to Ryan Chahrour, Zongbo Huang, B. Ravikumar, Ji Shen, Jian Wang, Pengfei Wang, Yicheng Wang, Changhua Yu, and Bo Zhao for their feedback. We thank participants at SED Annual Meeting, CICM, the Federal Reserve Bank of St. Louis, Tsinghua University, National School of Development at Peking University, Peking University HSBC Business School and CUHK-Shenzhen, and Central University of Finance and Economics for useful discussions and comments. We also thank Haoning Sun for excellent research assistance. Feng Dong acknowledges the financial support from the National Natural Science Foundation of China (#72250064, #72122011).

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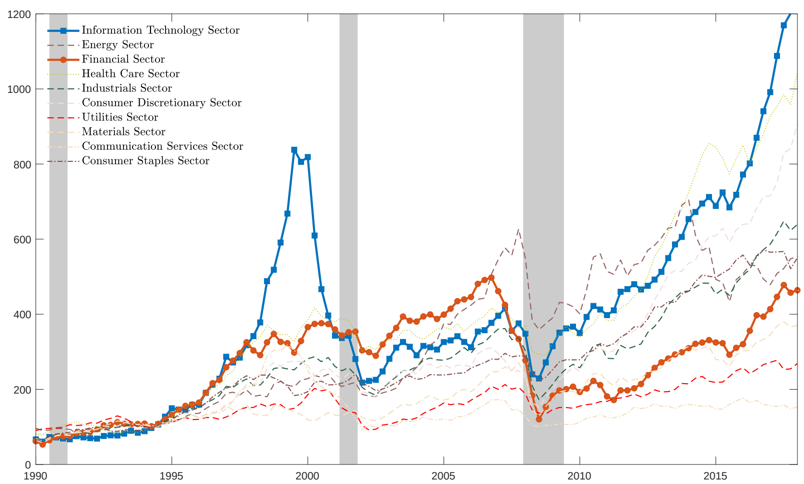
1. Introduction

A modern industrial economy is based on at least two fundamentally different network platforms: the production network and the financial network. The former creates an interdependence of tangible goods, characterized by the flow of materials across industries, and the latter creates an interdependence of intangible values, characterized by the flow of financial credit across industries. The second network does not necessarily mirror the first in the pattern of exchange. For example, although every industry uses information technology (IT) or goods from the IT industry, their financing and stock prices may not depend heavily on the IT industry. In fact, Figure 1 shows that the collapse of the IT industry's stock price in 2000, which many believe to be a bubble burst, did not cause a significant decline in the stock prices of other industries; in contrast, the decline in the financial industry's stock price around 2007 caused a significant decline in the stock prices of virtually all other industries during the Great Recession of 2008.

Figure 1: US Stock Prices By Industries



To rationalize the different interdependence and transmission mechanisms for stock prices in the financial network from those in the production network, as well as their dramatically different impact on the overall economy, we propose a framework for studying the contagiousness of stock price bubbles in a multi-industry economy with financial linkages across industries. We show that rational stock price bubbles can occur in different industries and can be contagious–that is, the bursting of a stock price bubble in one industry can trigger the bursting of stock price bubbles in other industries, regardless of the industry's importance in the production network. We provide a complete characterization of the conditions under which such bubble contagion occurs. In a calibrated version of the model, we show that, due to the strong dependence of firm values on credit flows in the financial network, the bursting of the stock price bubble in the financial industry can indeed burst the stock price bubbles in other industries.

Stock price bubbles arise in our model due to the presence of financial frictions in the form of borrowing constraints, à la Gomes, Jermann, and Schmid (2016) and Miao and Wang (2018). In particular, firms accumulate capital but are subject to an idiosyncratic shock to the rate of return on capital investment—the so-called investment efficiency shock.[1] Firms' investment returns are more efficient when the shock is larger. In the absence of borrowing constraints, only the firm receiving the largest investment efficiency shock should undertake investment by borrowing from all other firms, while these less efficient firms should choose to become lenders. However, with financial frictions, only a limited amount of borrowing is allowed; thus, some inefficient firms also choose to invest as long as their investment efficiency shock exceeds a cutoff value—which is an endogenously determined variable in the general equilibrium of the credit market. Since a firm's equity value can serve as collateral for borrowing, higher perceived firm values (stock prices) in a particular industry can increase that industry's borrowing capacity, thereby increasing the overall investment efficiency and profitability of firms in that industry, which in turn justifies the favorable perception of firm values in the first place. This creates a self-fulfilling industry-specific stock bubble. In such a bubble equilibrium, bubbles allow firms to better overcome financial frictions, and the industry thus achieves a more efficient allocation.

The key to our analysis is the spillover effects of stock price bubbles across industries through a financial network, where the liquidation value of a firm depends on the perceptions of other firms in all industries.[2] In particular, a firm's value at default depends in part on its acquisition price or potential fire-sale price from the perspective (or animal spirits) of potential buyers across the economy. As a result, the extent to which a firm can borrow depends not only on its own equity value, but also on the equity values of other firms in different industries. The financial linkage is modelled as an acquisition matrix, which measures the probability that a firm in one industry will be acquired by firms in another industry in the event of debt default. It follows that both the liquidation value and the borrowing limit of firms in one industry may be relaxed when firms in other industries are performing well, either due to improved fundamentals or the presence of larger industry-specific bubbles. This type of financial linkage naturally makes rational bubbles in different industries interdependent and thus leaves room for bubble contagion.

The main contribution of the paper to the existing literature is to provide a complete characterization of the entire set of bubble equilibria in a multi-industry economy, and to provide the condition under which the bursting of a bubble in one industry becomes contagious to other industries. It turns out that in a multi-industry model, bubbles occur in any subset of industries only if the implicit efficiency cutoff in those industries is higher than that in the bubbleless equilibrium. This endogenous efficiency cutoff is a function of financial linkages across industries and the severity of financial fric-

[1]See Wang and Wen (2012b) for example.

[2]There are many ways to model financial linkages. For simplicity and ease of calibration, we model them as flows of firm value through acquisition.

tions, such that industrial bubbles are more likely to occur when financial linkages are strong, or when equity collateralization is high, or when capital-specific collateralization is low. Intuitively, bubbles in different industries complement each other and are easier to sustain when a larger set of industries contains bubbles. Such interdependence can be reinforced by higher equity pledgeability. More subtly, bubbles are harder to sustain or survive under higher capital pledgeability because physical capital is more useful in easing borrowing constraints and thus dampening the need for bubbles. In the end, thanks to the Perron-Frobenius theorem, industrial bubbles boil down to an eigenvector of the capital-pledgeability-weighted financial linkages matrix.

This theoretical characterization allows us to identify the conditions for bubble contagion. We demonstrate the bubble contagion mechanism in our model by considering an industry to be critical if a bubble bursting in that industry necessarily causes bubbles in some other industries to burst at the same time. We find that an industry is critical if other industries are highly financially dependent on that industry through the financial network. In short, there is a critical industry that will induce bubble contagion if the assets of firms in other industries are more likely to shift to that industry in the event of default–namely, a critical industry must be the major holder of firm equity in the economy. The aforementioned intuition on pledgeability also implies that the financial system is more vulnerable to the risk of bubble contagion when equity pledgeability is high or capital-specific pledgeability is low. Our result on critical industries is consistent with the insight of Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) who argue that the systemically important financial institutions are those with closer financial linkages to others.[3]

In our framework, financial linkages play a different role from standard input-output linkages in shaping the properties of bubbles. Specifically, in a bubble equilibrium, the production network only affects the average level of bubbles but is independent of the relative size of bubbles across industries, while both the former and the latter depend on financial linkages. Moreover, an industry that is critical in generating bubbles may not be important in generating value added or total sales. These differences call for additional attention to the identification of intangible financial linkages.

To this end, we use U.S. merger and acquisition transaction data to calibrate the financial linkage matrix in our model. The propensity of a firm in one industry to acquire or merge with firms in another industry reflects the underlying financial linkages that our theory addresses. It turns out that the financial industry is the most important one of the financial linkages in the sense that firms in the manufacturing and services industries are more likely to be acquired or merged with by firms in the financial industry, but not vice versa.

According to our calibration, the economy exhibits unidirectional bubble contagion: the bursting of bubbles in the financial industry drags the whole economy down through the bursting of bubbles in

[3]Note that the financial linkage in our model differs from that in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), where they emphasise the debt linkage between different banks. Instead, we focus on the linkage between industries.

other industries, while industrial bubbles remain intact after the bursting of bubbles in manufacturing or services. The changes in output reflect the pattern of bubble contagion. The output loss is much more severe when contagion occurs. In contrast, the overall economy is only slightly affected after the bursting of bubbles in manufacturing.

Bubble contagion introduces an additional source of aggregate uncertainty, while at the same time bubbles can help stabilize the economy in response to traditional financial shocks. An increase in capital pledgeability leads to a boom in the bubble equilibrium as financial constraints are relaxed. In a bubble equilibrium, the increase in capital pledgeability reduces the size of bubbles or even bursts them. Consequently, the relaxation of financial constraints during a boom is more limited than in the bubbleless economy. At first glance, this channel may look similar to the traditional automatic stabilizer via the fiscal channel (Gali, 1994; McKay and Reis, 2016), but it is important to recognize that in our setting such a stabilizing role is shock specific and bubbles may amplify the effects of other disturbances such as TFP shocks.

Related literature. The theoretical literature on bubbles is extensive.[4] Our paper builds on the large body of works on rational bubbles (Arce and López-Salido, 2011; Martin and Ventura, 2012; Farhi and Tirole, 2011; Galí, 2014; Martin and Ventura, 2016; Ikeda and Phan, 2016; Asriyan et al., 2016; Bengui and Phan, 2018; Biswas, Hanson, and Phan, 2020; Ikeda and Phan, 2019; Dong, Miao, and Wang, 2020). Our paper is most closely related to the research based on firm bubbles in an infinite-horizon framework (Kocherlakota, 2009; Wang and Wen, 2012b; Miao and Wang, 2014; Hirano, Inaba, and Yanagawa, 2015; Miao, Wang, and Xu, 2015; Hirano and Yanagawa, 2016; Miao and Wang, 2018). Our paper contributes to the literature by introducing and characterizing contagious bubbles in a multi-industry economy, and how bubbles modify the response of the economy to various shocks.

Our work is also closely related to the growing literature on financial networks and systemic risk (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Cabrales, Gottardi, and Vega-Redondo, 2017; Kopytov, 2018; Jackson and Pernoud, 2021; Denbee et al., 2021). Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Denbee et al. (2021) also point out that there are some key players in the financial network whose failure is more likely to generate systemic risk. Huremovic et al. (2020) study the interaction between the production network and the financial network, and show that banking shocks can propagate upstream and downstream along the production network, amplifying the impact of banking shocks. Our results complement this line of research by showing that systematic risk may also be present in asset prices that are not fundamental in the conventional sense.

[4]See Miao (2014) and Martin and Ventura (2018) for comprehensive surveys of rational bubbles.

2. Model Environment

Time is discrete. There are two types of firms: intermediate goods firms and final goods firms. All firms are owned by households. There are $S > 1$ industries for intermediate goods production, and each industry $j \in \{1, 2, \ldots, S\}$ has a unit measure of infinitely many firms indexed by $\iota \in [0, 1]$.

2.1 Households

To simplify the model structure, we assume that there is a representative banking household (owners of banks) that trades in corporate equities (stocks), makes/accepts deposits, and supplies labor to various industries. The household's problem is given by

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t\right), \tag{2.1}$$

subject to

$$C_t + \sum_{j=1}^{S} \int_0^1 s_{j,t+1}\left(\iota\right)\left(V_{jt}\left(\iota\right) - d_{jt}\left(\iota\right)\right) \mathrm{d}\iota + \frac{D_{t+1}}{1+r_t} = \sum_{j=1}^{S} \int_0^1 s_{jt}\left(\iota\right) V_{jt}\left(\iota\right) \mathrm{d}\iota + W_t + D_t,$$

and

$$D_{t+1} \geq 0.$$

where β is the discount factor, C_t is consumption, D_t is the deposit, and r_t is the interest rate. Later, we will also use the discount rate,

$$\chi \equiv \frac{1}{\beta} - 1,$$

to characterize the equilibrium. We assume that labor is perfectly mobile across industries and that total labor supply is normalized to one.[5] In terms of household portfolio choice, let $s_{jt}\left(\iota\right)$ denote the stock holdings of firm ι's equity share in industry j at period t, $V_{jt}\left(\iota\right)$ denote the date-t stock price of firm ι in industry j, and $d_{jt}\left(\iota\right)$ denote the dividend associated with holding stock $V_{jt}\left(\iota\right)$. For later use, we define $\Lambda_t \equiv u'\left(C_t\right)$ as the marginal utility of consumption.

[5] The assumption of inelastic labor supply helps to simplify the characterization of the steady-state allocation, and we can endogenize labor supply without changing the main results.

2.2 Final Goods Firms

A large number of perfectly competitive firms transform industrial output X_{jt} into a final consumption and investment good Y_t, with a Cobb-Douglas aggregator

$$Y_t = \prod_{j=1}^{S} X_{jt}^{\varphi_j}.$$

We normalize the final goods price to one, and it follows that firms solve the following problem

$$\max_{\{X_{jt}\}} \; Y_t - \sum_{j=1}^{S} P_{jt} X_{jt}. \tag{2.2}$$

Optimal demand implies that the ratio of output of industry j to total GDP (Gross Domestic Product) is determined by:

$$P_{jt} = \varphi_j \frac{Y_t}{X_{jt}}. \tag{2.3}$$

2.3 Intermediate Goods Firms

There is a unit measure of firms in each industry j. The production function of firm ι in industry j is given by

$$o_{jt}(\iota) = k_{jt}^{\alpha}(\iota)\, n_{jt}^{1-\alpha}(\iota), \tag{2.4}$$

where $k_{it}(\iota)$ is a given capital stock and $n_{it}(\iota)$ is labor input.[6] Given the capital stock, firm ι chooses its optimal employment level. Due to constant returns to scale, revenue less labor costs is proportional to the firm's capital stock:

$$R_{jt} k_{jt}(\iota) \equiv \max_{n_{jt}(\iota)} P_{jt} o_{jt}(\iota) - W_t n_{jt}(\iota), \tag{2.5}$$

where $R_{jt} \equiv \left(\frac{1-\alpha}{W_t}\right)^{\frac{1-\alpha}{\alpha}}$ is the marginal product of capital.

To accumulate capital, a firm chooses the rate of investment $i_{jt}(\iota)$ in each period under the irreversibility condition:

$$i_{jt}(\iota) \geq 0. \tag{2.6}$$

Since capital goods are the same as consumer goods, their price is 1. As in Wang and Wen (2012b), the law of motion of capital accumulation is subject to an idiosyncratic investment efficiency shock $\epsilon_{jt}(\iota)$:

$$k_{j,t+1}(\iota) = (1-\delta)\, k_{jt}(\iota) + \epsilon_{jt}(\iota)\, i_{jt}(\iota), \tag{2.7}$$

where δ is the depreciation rate and $\epsilon_{jt}(\iota)$ the investment efficiency shock to firm ι in industry j. We

[6] In the baseline analysis, we assume that intermediate goods production only requires primary inputs.

assume that $\epsilon_{jt}(\iota)$ is i.i.d. across firms and industries and over time, with CDF $F(\epsilon)$ and support $(\underline{\epsilon}, \overline{\epsilon})$.[7] The mean of the efficiency shock is normalized to $\mathbb{E}[\epsilon] = 1$.

A firm's investment can be financed by an internal source and an external source: the internal source of financing is the revenue $R_{jt}(\iota)\, k_{jt}(\iota)$ and the external source of financing is the issuance of debt $l_{j,t+1}(\iota)$. Given the capital stock $k_{jt}(\iota)$, the existing amount of debt $l_{jt}(\iota)$, and the investment efficiency shock $\epsilon_{jt}(\iota)$ realized at the beginning of each period, the firm's problem is to choose an investment path that maximizes its stock market value or the present value of dividends $d_{jt}(i)$:

$$V_{jt}\left(k_{jt}(\iota), l_{jt}(\iota), \epsilon_{jt}(\iota)\right) = \max_{\{k_{j,t+1}(\iota), l_{j,t+1}(\iota)\}} d_{jt}(\iota) + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(k_{j,t+1}(\iota), l_{j,t+1}(\iota), \epsilon_{j,t+1}(\iota)\right),$$

where the current dividend $d_{jt}(\iota)$ is given by

$$d_{jt}(\iota) = R_{jt} k_{jt}(\iota) + \frac{1}{1+r_t} l_{j,t+1}(\iota) - l_{jt}(\iota) - i_{jt}(\iota). \tag{2.8}$$

Clearly, firms that receive good investment efficiency shocks would like to borrow from outside—households, banks, or firms—by issuing debt $l_{j,t+1}(\iota)$, but the amount of new debt issuance $l_{j,t+1}(\iota)$ is subject to financial constraints, which we discuss in detail next.

Financial frictions. The external financing market is imperfect, so firms in industry j face the following constraints:[8]

$$d_{jt}(\iota) \geq 0, \tag{2.9}$$

$$\frac{l_{,j,t+1}(\iota)}{1+r_t} \leq \xi_j \sum_{i=1}^{S} \mathcal{M}_{ji} \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \overline{V}_{i,t+1}\left(\sigma_j k_{jt}(\iota), 0\right). \tag{2.10}$$

Constraint (2.9) implies that the firm cannot borrow directly from its owners by paying them negative dividends. Constraint (2.10) implies that debt issuance is limited due to default risk, where $\overline{V}_{j,t+1}\left(k_{j,t+1}(\iota), l_{j,t+1}(\iota)\right) \equiv \int V_{t+1}\left(k_{j,t+1}(\iota), l_{j,t+1}(\iota), \epsilon_{j,t+1}(\iota)\right) \mathrm{d}F\left(\epsilon_{j,t+1}(\iota)\right)$ is the expected value of a firm in the next period before the idiosyncratic investment shock is realized, which serves as part of the "collateral value" in the case of default.[9] In particular, creditors are only able to recover a $\sigma_j \in (0,1)$ fraction of the installed capital k_{jt}, and they can merge the firm with another firm in industry i with acquisition probability $\mathcal{M}_{ji}$. Due to the fire-sale risk, the value of the firm is also discounted by an industry-specific factor $\xi_j \in (0,1)$.

[7] Tractability is well preserved if the distribution is industry specific: $F_j(\cdot)$.

[8] A more generalized setup for (2.9) is $d_t(\iota) \geq -\nu \cdot k_t(\iota)$, where $\nu \geq 0$ governs the severity of equity frictions. Without loss of generality, we set $\nu = 0$ to simplify our analysis. See Wang and Wen (2012a) and Miao, Wang, and Xu (2015), for details of the generalized setup.

[9] See Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Jermann and Quadrini (2012) and Moll (2014) etc. for the discussion of borrowing constraints and financial accelerators.

An equivalent form of the borrowing constraint (2.10) is the following incentive constraint:

$$\begin{aligned}\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\overline{V}_{j,t+1}\left(k_{j,t+1}(\iota), l_{j,t+1}(\iota)\right) \quad \geq \quad & \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\overline{V}_{j,t+1}\left(k_{j,t+1}(\iota), 0\right) \\ & -\xi_j \sum_{i=1}^{S} \mathcal{M}_{ji}\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\overline{V}_{i,t+1}\left(\sigma_j k_{jt}(\iota), 0\right),\end{aligned} \tag{2.11}$$

which is a multi-industry extension of the incentive constraint à la Miao and Wang (2018). That is, the debt constraint (2.10) can be interpreted as an incentive constraint in the presence of limited firm commitment: Firm ι in industry j decides at the beginning of $t+1$ whether to default on its debt; without default, the continuation value is simply the expected value of operating on the LHS of condition (2.11), where new debt issuance $l_{j,t+1}$ is allowed; in the case of default, the debt is renegotiated and no new debt issuance is allowed: $l_{j,t+1} = 0$. In this second scenario, the creditors are able to confiscate or recover $\sigma_j k_{jt}$ units of capital and rebuild a firm or merge it with another firm in a different industry i with acquisition probability $\mathcal{M}_{ji}$. Due to the mismatch or fire sale, the value of the firm is also discounted by ξ_j in the event of default. The parameter space $\{\sigma_j, \mathcal{M}_{ji}, \xi_j\}$ thus captures the severity of financial frictions as well as financial linkages through the acquisition probability matrix $\mathcal{M}_{ji}$.

In other words, because a defaulting firm can be merged or matched with a firm in another industry, the value of the firm (stock price) has spillover effects on the financial conditions of other industries, which in turn affect the level of aggregate investment. Moreover, as we explain in the next section, the interdependence of credit limits also helps to either transmit or block industrial bubbles, which is the core of our analysis.

We define $\theta_j^k \equiv \xi_j \sigma_j$ and $\theta_j^b \equiv \xi_j$, which can be interpreted as capital pledgeability and equity pledgeability, respectively. Changes in these two parameters can be interpreted as credit shocks (Jermann and Quadrini, 2012).

2.4 Industry-Specific Bubbles

In this section, we characterize the value function of firms and their investment decisions. We show the possibility that the value function of a firm ι in industry j contains an industry-specific bubble that is independent of fundamentals. In the next section, we provide conditions under which such a non-fundamental component can persist in equilibrium in different industries and become contagious across industries.

Intuitively, firms with better investment efficiency are willing to invest more. In fact, due to the CRS assumption and homogeneous production technology, the most efficient allocation is to have the firm with the highest efficiency shock $\epsilon_{jt}(\iota)$ in industry j make all the investments by borrowing from other firms in the economy. However, this allocation is not feasible because of the borrowing constraint

(2.10). Thus, in equilibrium, firms in industry j with efficiency shocks higher than an industry-specific threshold ϵ^*_{jt} will borrow up to their borrowing limits to invest, while the remaining firms will choose to save and not invest.

The industry-specific cutoff ϵ^*_{jt} is thus a measure of efficiency and crucial for understanding the equilibrium properties. A higher cutoff value implies a more efficient allocation or a lower degree of investment misallocation in the industry. We denote the degree of investment misallocation in industry j as

$$\Gamma\left(\epsilon^*_{jt}\right) \equiv \int_{\epsilon^*_{jt}}^{\overline{\epsilon}} \left(\frac{\epsilon}{\epsilon^*_{jt}} - 1\right) \mathrm{d}F, \tag{2.12}$$

where $\Gamma(\cdot)$ decreases with ϵ^*_{jt}. For example, as ϵ^*_{jt} approaches the upper bound $\overline{\epsilon}$, $\Gamma\left(\epsilon^*_{jt}\right)$ vanishes to zero.

On the other hand, the tightness of the borrowing constraint (2.10) controls how far the equilibrium allocation is from the efficient one due to investment misallocation. If a firm's equity is perceived to be more valuable, such a belief may drive up the value of equity and relax the firm's borrowing constraint. This, in turn, may justify the optimistic perception in the first place. Thus, similar to Miao and Wang (2018), self-fulfilling bubbles can arise in equilibrium. In contrast to their work, the bubbles in our model are industry-specific and are contagious. The following proposition characterizes the dynamics that industrial bubbles must obey.

Proposition 2.1. *Let $B_t \geq 0$ be the non-fundamental value or bubble contained in a firm's stock price V_t. Whether or not there is a bubble in industry j, the following must hold:*

1. *The industry-specific efficiency cutoff is determined by $\epsilon^*_{jt} = \frac{1}{Q_{jt}}$.*
2. *The expected value of the firm in industry j is given by*

$$\mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} \overline{V}_{j,t+1}\left(k_{j,t+1}, l_{j,t+1}\right) = Q_{jt} k_{j,t+1} - \frac{l_{j,t+1}}{1+r_t} + B_{jt}. \tag{2.13}$$

3. *The industrial Tobin Q (Q_{jt}) satisfies*

$$Q_{jt} = \mathbb{E}_t \sum_{h=1}^{\infty} \left[\beta\left(1-\delta\right)\right]^{h-1} \frac{\beta\Lambda_{t+h}}{\Lambda_t} \left(R_{j,t+h}\left(1+\Gamma(\epsilon^*_{j,t+h})\right) + \theta^k_j \Gamma(\epsilon^*_{j,t+h}) \sum_{i=1}^{S} \mathcal{M}_{ji} Q_{i,t+h}\right). \tag{2.14}$$

4. *The law of motion for industrial bubble B_{jt} satisfies*

$$B_{jt} = \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} \left[B_{j,t+1} + \theta^b_j \Gamma(\epsilon^*_{j,t+1}) \sum_{i=1}^{S} \mathcal{M}_{ji} B_{i,t+1}\right], \quad \text{if } B_{jt} > 0. \tag{2.15}$$

5. *The borrowing constraint (2.11) can be rewritten as*

$$\frac{l_{j,t+1}(\iota)}{1+r_t} \leq \theta_j^k k_{jt}(\iota) \sum_{i=1}^{S} \mathcal{M}_{ji} Q_{it} + \theta_j^b \sum_{i=1}^{S} \mathcal{M}_{ji} B_{it}. \tag{2.16}$$

Point (1) states that the cutoff ϵ_{jt}^* is determined by the inverse of Tobin's Q. Namely, to increase one unit of capital for firm ι, the cost is $1/\epsilon_{jt}(\iota)$, and the return is Q_{jt}. It is only profitable to invest if $\epsilon_{jt}(\iota) > 1/Q_{jt} = \epsilon_{jt}^*$.

Point (2) states that the expected value of a firm in industry j depends on the industry-specific market value of its newly installed capital stock, $Q_{jt} k_{j,t+1}(\iota)$, minus its debt issuance, $\frac{l_{j,t+1}}{1+r_t}$, which can be interpreted as the fundamental valuation. In addition, the value of the firm may be higher if there is an industrial bubble B_{jt}.

Point (3) states that Tobin's Q is the present value of expected future capital returns, so it obeys an otherwise standard forward-looking Euler condition (2.14), except for the additional components that capture investment inefficiency and the collateral value in the financial network. For example, industrial investment is at its most efficient level when the value of Tobin's Q is at its minimum where $\Gamma_j\left(\epsilon_{jt}^*\right) = 0$.

Point (4) states that the bubble is self-fulfilling: Coordinating to $B_{jt} = 0$ for all j always satisfies this forward-looking Euler equation, while the perception of positive bubbles in some industries could also be consistent with rational expectations.

The conditions (2.14) and (2.15) imply that both the Tobin's Q and the industrial bubbles tend to increase with the misallocation measure $\Gamma_j\left(\epsilon_{jt}^*\right)$. A larger $\Gamma_j\left(\epsilon_{jt}^*\right)$ requires additional compensation for individual investment risk by holding capital, and it induces a stronger demand for bubbles to overcome the financial friction.

Point (5) makes it clear that industrial bubbles help to overcome financial frictions. The presence of bubbles increases the equity value of firms, the importance of which is determined by the equity pledgeability θ_j^b and the financial linkages.

Once the investment efficiency cutoffs are determined, the industrial capital and investment can be solved accordingly, as shown below:

Corollary 1. *The optimal total investment I_{jt} and law of motion of industrial capital K_{jt} are given by*

$$\begin{aligned} I_{jt} &= \left[R_{jt}K_{jt} + \theta_j^k \sum_{i=1}^{S} \mathcal{M}_{ji} Q_{it} K_{jt} + \theta_j^b \sum_{i=1}^{S} \mathcal{M}_{ji} B_{it} - L_{jt}\right] \cdot \left[1 - F\left(\epsilon_{jt}^*\right)\right], && (2.17) \\ K_{j,t+1} &= (1-\delta) K_{jt} + \mathbb{E}\left(\epsilon | \epsilon \geq \epsilon_{jt}^*\right) \cdot I_{jt}. && (2.18) \end{aligned}$$

It is useful to note that the cutoff value affects the capital accumulation both through the fraction of active investing firms $\left[1 - F\left(\epsilon^*_{jt}\right)\right]$ and through the average investment efficiency $\mathbb{E}\left(\epsilon_j|\epsilon_j \geq \epsilon^*_{jt}\right)$.

3. Equilibrium Characterization

In the baseline model, we focus on the case without aggregate shocks. As a result, along the transition path with rational expectations, whether industrial bubbles can exist or not boils down to whether they can exist in the steady state. To clarify the terminology, we classify the equilibria according to their bubble characteristics. In particular, we allow for the possibility that bubbles can exist in some but not all industries.

Definition 1 (Bubbleless and Bubbly Equilibrium)**.** *In a bubbleless (i.e., fundamental) equilibrium, $B_j = 0$ for all industries $j \in \mathbf{S}$. In a $\boldsymbol{s}$-bubble equilibrium, $B_j > 0$ for $j \in \boldsymbol{s} \subset \mathcal{P}(\mathbf{S})$, where $\mathcal{P}(\mathbf{S})$ denotes the power subset of $\mathbf{S} = \{1, ..., S\}$.*

3.1 Determination of Bubbles

As suggested in the previous analysis, the cutoff values for the investment decision govern the capital accumulation process. In the steady state, regardless of whether there are bubbles or not, the following lemma shows that the cutoffs are equalized across industries.

Lemma 1. *In the steady state, the Tobin's Q and the cutoff values are equalized across industries,*

$$\epsilon_j = \epsilon^* = \frac{1}{Q(\epsilon^*)},$$

and the interest rate on loan is given by

$$r = \frac{\chi - \Gamma(\epsilon^*)}{1 + \Gamma(\epsilon^*)}, \qquad \textit{where} \quad \chi \equiv \frac{1}{\beta} - 1.$$

The interest rate increases in the single (universal) efficiency cutoff ϵ^* as investment risk decreases. In particular, as the misallocation measure approaches zero, the interest rate simply equals the discount rate, $r = \chi$. In the following, we will show that this universal efficiency cutoff is also the key determinant of whether bubbles can survive in equilibrium.

When firms finance their investments, their loans must be backed by the value of their collateral. Let $\Phi(\epsilon)$ be the excess demand function for credit in the absence of bubbles. To facilitate the characterization, we make the following assumption about the primitives of the model to ensure that the excess credit demand function for credit is well behaved.

Assumption. *The excess credit demand function $\Phi(x)$ is strictly increasing in x, where*

$$\Phi(x) = \sum_{j=1}^{S} \frac{\alpha \varphi_j}{\chi + \delta - \theta_j^k \Gamma(x)} \left(\frac{\delta x F(x)}{\int_{\varepsilon > x} \varepsilon dF} - \chi - \theta_j^k \right). \tag{3.1}$$

Since $\frac{xF(x)}{\int_{\varepsilon > x} \varepsilon dF}$ is increasing in x, this assumption is always satisfied when θ_j^k is relatively small. We also provide the characterization of the equilibrium when this assumption is not satisfied, although this is not the case in our quantitative exercise.

3.1.1 Bubbleless Equilibrium

First, consider the bubbleless equilibrium, where $B_j = 0$ for all j. Clearly, the excess demand for credit must be zero, which pins down the efficiency cutoff.

Proposition 3.1 (Bubbleless Equilibrium)**.** *There exists a unique bubbleless equilibrium. The efficiency cutoff ϵ^f in the bubbleless equilibrium satisfies*

$$\Phi(\epsilon^f) = 0, \tag{3.2}$$

where $\Phi(x)$ is defined in equation (3.1).

Without a bubble, condition (3.2) implies that ϵ_f^* is determined by the capital pledgeability vector $\{\theta_1^k, \ldots, \theta_S^k\}$. In the special case where $\theta_j^k = \theta^k$, this condition reduces to

$$\frac{\delta \epsilon^f F\left(\epsilon^f\right)}{\int_{\varepsilon > \varepsilon^f} \varepsilon dF} - \chi = \theta^k, \tag{3.3}$$

where it is easy to verify that the efficiency cutoff ϵ^f increases in θ^k. Intuitively, a higher θ^k relaxes the financial constraint, which in turn helps to mitigate the investment misallocation. In the general case with heterogeneous θ_j^k, the efficiency cutoff ϵ^f is determined by a weighted average of the capital pledgeability across industries.

3.1.2 Bubble Equilibrium

Now consider the case where bubbles occur in a subset of industries, $\boldsymbol{s} \subset \mathcal{P}(\mathbf{S})$. Let $\mathcal{M}_{\boldsymbol{s}}$ be the financial linkage matrix $\mathcal{M}$ which keeps columns and rows in $\boldsymbol{s}$, $\mathbf{B}_{\boldsymbol{s}}$ the vector of bubbles, and $\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^b\right)$ the matrix which keeps the equity parameter θ_j^b on the diagonal for $j \in \boldsymbol{s}$. For any industry $i, j \in \boldsymbol{s} \subset \mathcal{P}(\mathbf{S})$, the Euler equation in (2.15) can be rewritten as

$$\frac{\chi}{\Gamma(\epsilon^*)} B_i = \theta_i^b \sum_{j \in \boldsymbol{s}} \mathcal{M}_{ij} B_j. \tag{3.4}$$

More compactly, a industrial bubble must satisfy

$$\left(\frac{\chi}{\Gamma\left(\epsilon^{*}\right)}\mathbf{I}-\operatorname{diag}(\boldsymbol{\theta}_{\boldsymbol{s}}^{b})\mathcal{M}_{\boldsymbol{s}}\right)\mathbf{B}_{\boldsymbol{s}}=\mathbf{0}. \tag{3.5}$$

The existence condition of the $\boldsymbol{s}$-bubble equilibrium is characterized below:

Proposition 3.2 (Bubble Equilibrium)**.** *If the stochastic acquisition matrix $\mathcal{M}$ is irreducible, then for a selection of industries $\boldsymbol{s}\subset\mathcal{P}\left(\mathbf{S}\right)$, there exists a unique $\boldsymbol{s}$-bubble equilibrium if and only if*

$$\rho\left(\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^{b}\right)\mathcal{M}_{\boldsymbol{s}}\right)>\frac{\chi}{\Gamma\left(\epsilon^{f}\right)}, \tag{3.6}$$

where $\rho\left(\cdot\right)$ selects the leading eigenvalue.[10] *The associated efficiency cutoff $\epsilon_{\boldsymbol{s}}^{b}$ is given by*

$$\epsilon_{\boldsymbol{s}}^{b}=\Gamma^{-1}\left(\frac{\chi}{\rho\left(\operatorname{diag}(\boldsymbol{\theta}_{\boldsymbol{s}}^{b})\mathcal{M}_{\boldsymbol{s}}\right)}\right)>\epsilon_{f}. \tag{3.7}$$

The inequality (3.6) gives the necessary and sufficient condition for the existence of the $\boldsymbol{s}$-bubble equilibrium. The basic logic is that for bubbles to exist, the corresponding efficiency cutoff must be high enough to exceed that of the bubbleless equilibrium. That is, the economy must leave room for bubbles to help overcome financial frictions and improve allocative efficiency. Technically, the condition (3.6) is obtained using the Perron-Robenius theorem, which guarantees the existence of industrial bubbles since elements in the eigenvector are positive in the linear system (3.5).

Figure 2: Bubbleless and Bubbly Equilibria

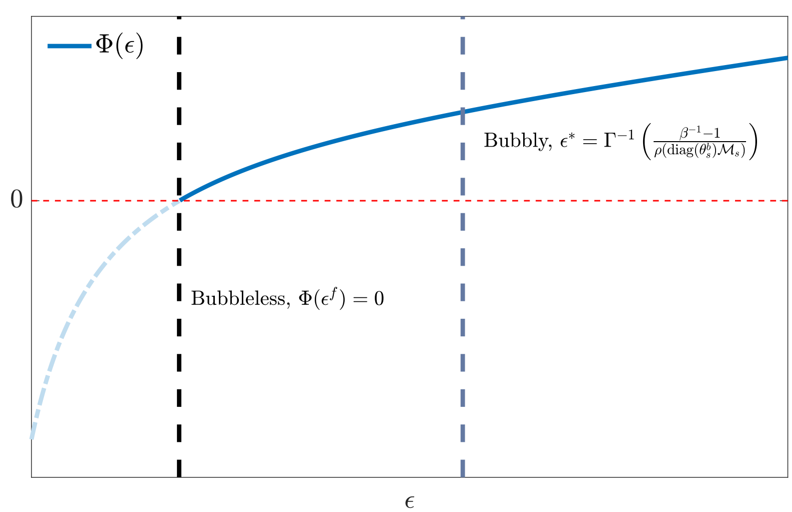


Figure 2 helps to visualize this logic. The blue line represents the excess credit demand function $\Phi(x)$. In the bubbleless equilibrium, the efficiency cutoff solves $\Phi(\epsilon^{f})=0$. But in the bubble equilib-

[10]With $\boldsymbol{\theta}_{\boldsymbol{s}}^{b}$ and $\mathcal{M}_{\boldsymbol{s}}$ being positive matrices, the eigenvalues are all positive.

rium, the efficiency cutoff solves a different condition: $\Gamma(\epsilon^*) = \frac{\chi}{\rho\left(\mathrm{diag}(\boldsymbol{\theta}^b_{\boldsymbol{s}})\mathcal{M}_{\boldsymbol{s}}\right)}$. The bubble equilibrium exists only if the cutoff is sufficiently high, or if the excess demand for credit is sufficiently high. Note also that $\Phi(\epsilon^*) > 0$, which means that additional credit can be supplied by bubbles.

But under what conditions are bubbles more likely to occur? First, note that both capital and equity pledgeability are important in determining the bubble equilibrium. On the one hand, the leading eigenvalue increases with equity pledgeability $\boldsymbol{\theta}^b_{\boldsymbol{s}}$, and bubbles are more likely to occur with larger $\boldsymbol{\theta}^b_s$. On the other hand, the efficiency cutoff in the bubbleless equilibrium increases with capital pledgeability $\boldsymbol{\theta}^k$, and bubbles are less likely to occur with larger $\boldsymbol{\theta}^k_s$. These two forces underscore the different roles played by equity and capital pledgeability: higher θ^k_j makes capital more valuable as collateral, reducing the need for bubbles; higher θ^b_j makes bubbles more valuable in facilitating borrowing, increasing the need for bubbles.

That is, holding $\boldsymbol{\theta}^k$ fixed, a universal improvement in equity pledgeability always enlarges the parameter space for bubble equilibrium.

Corollary 2. *Consider relaxing stock pledgeability $\widetilde{\boldsymbol{\theta}}^b_{\boldsymbol{s}}$, i.e., $\widetilde{\theta}^b_i \geq \theta^b_i$ for all $i \in \boldsymbol{s}$ and $\widetilde{\theta}^b_j > \theta^b_j$ for some $j \in \boldsymbol{s}$. If the $\boldsymbol{s}$-bubble equilibrium exists under $\boldsymbol{\theta}^b_{\boldsymbol{s}}$, then it also exists under $\widetilde{\boldsymbol{\theta}}^b_{\boldsymbol{s}}$, but not vice versa.*

Second, the structure of the financial linkages $\mathcal{M}$ naturally shapes the existence of the bubble equilibrium. The following corollary further provides a more intuitive estimate of the likelihood of the bubble equilibrium. In general, a more connected group of industries is more likely to allow industrial bubbles to exist. An industry with almost no connection to other industries eliminates the possibility of bubbles.

Corollary 3. *The dominant eigenvalue of $\rho\left(\mathrm{diag}\left(\boldsymbol{\theta}^b_{\boldsymbol{s}}\right)\mathcal{M}_{\boldsymbol{s}}\right)$ is bounded by*

$$\rho\left(\mathrm{diag}\left(\boldsymbol{\theta}^b_{\boldsymbol{s}}\right)\mathcal{M}_{\boldsymbol{s}}\right) \in \left[\min_{i\in\boldsymbol{s}}\sum_{j\in\boldsymbol{s}}\theta^b_i\mathcal{M}_{ij}, \max_{i\in\boldsymbol{s}}\sum_{j\in\boldsymbol{s}}\theta^b_i\mathcal{M}_{ij}\right], \tag{3.8}$$

and the $\boldsymbol{s}$-bubble equilibrium exists if $\min_{i\in\boldsymbol{s}}\sum_{j\in\boldsymbol{s}}\theta^b_i\phi_{ij} > \frac{\chi}{\Gamma(\epsilon^f)}$.

So far we have focused on one particular $\boldsymbol{s}$-bubble equilibrium. Another question of interest is, among many different groups of industries, which groups are more likely to sustain industrial bubbles? The following corollary answers this question.

Corollary 4. *For any $\boldsymbol{s}\subset\boldsymbol{s}'\subset\mathcal{P}(\mathbf{S})$, if the $\boldsymbol{s}$-bubble equilibrium exists, then the bubble equilibrium s' can also be sustained with $\epsilon^b_{\boldsymbol{s}} < \epsilon^b_{\boldsymbol{s}'}$.*

That is, if a small group can sustain bubbles, then a strictly larger group can sustain bubbles with a higher efficiency cutoff. In particular, if one is wondering whether bubbles can exist at all, then it is sufficient to compare the leading eigenvalue of the matrix $\mathrm{diag}\left(\boldsymbol{\theta}^b\right)\mathcal{M}$ associated with the entire economy with $\frac{\chi}{\Gamma(\epsilon^f)}$.[11] In Section 4, we visit the reverse question of whether bubbles can be contagious, in the sense that the bursting of the bubble in industry j can destroy the bubbles in its connected industries.

Moreover, the above corollary implies that the upper bound of the efficiency cutoff is given by $\epsilon_{\mathbf{S}}^b$, which in turn is determined by

$$\rho\left(\operatorname{diag}\left(\boldsymbol{\theta}_{\mathbf{S}}^b\right)\mathcal{M}_{\mathbf{S}}\right)=\frac{\chi}{\Gamma\left(\epsilon_{\mathbf{S}}^b\right)}. \tag{3.9}$$

In particular, if $\theta_j^b=\theta^b$ for all j, then the above equation can be further simplified to $\theta^b=\frac{\chi}{\Gamma(\epsilon_{\mathbf{S}}^b)}$. So $\epsilon_{\mathbf{S}}^b=\Gamma^{-1}\left(\frac{\chi}{\theta^b}\right)$, which strictly increases with θ^b.

3.2 Resource Allocation

In this subsection, we illustrate how the equilibrium allocation depends on the efficiency cutoff. In the absence of financial frictions, only firms that receive the highest shock invest. In the presence of financial frictions, it is tempting to conclude that a higher efficiency cutoff monotonically brings outcomes closer to the frictionless benchmark. However, the equilibrium allocation actually depends on the efficiency cutoff in a more subtle way.

Proposition 3.3 (Output)**.** *Given the efficiency cutoff ϵ^*, total output in both bubbleless and bubble equilibria is given by*

$$\log Y=\frac{\alpha}{1-\alpha}\left\{\log\epsilon^*+\log(1+\Gamma(\epsilon^*))-\sum_{j=1}^{S}\varphi_j\log\left(\chi+\delta-\theta_j^k\Gamma(\epsilon^*)\right)\right\}+\mathtt{constant}. \tag{3.10}$$

Condition (3.10) shows that the efficiency cutoff can affect aggregate output through several different channels. First, the direct effect of a higher ϵ^* is to improve allocation efficiency, which facilitates capital accumulation and output expansion. Second, recall from Lemma 1 that a higher ϵ^* implies a higher risk-free rate. Behind this change is a weakened precautionary saving motive due to a reduction in uncertainty, which discourages capital accumulation. Third, a higher efficiency cutoff also reduces the collateral value of capital, making saving in capital less attractive. These competing forces paint a mixed picture of how output varies with the efficiency cutoff.[12]

Proposition 3.2 gives the existence condition for a s-bubble equilibrium. The following proposition further characterizes the size of bubbles in each industry.

[11] If $\theta_j^b=1$ for all $j\in\mathbf{S}$, then $\Gamma(\epsilon^*)=\chi$. This is exactly the bubble solution in the one-industry model developed by Wang and Wen (2012b).

[12] Our numerical analysis finds an inverted U-shaped relationship, with the latter two channels dominating when ϵ^* is relatively small, and the former channel dominating when ϵ^* is relatively large.

Proposition 3.4 (Industrial Bubble)**.** *In a $\boldsymbol{s}$-bubble equilibrium, the vector of bubbles $\{B_j\}$ for $j \in \boldsymbol{s}$ is a leading eigenvector of matrix* $\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^b\right) \mathcal{M}_{\boldsymbol{s}}$ *which satisfies*

$$\frac{\sum_{j \in \boldsymbol{s}} \theta_j^b \mathcal{M}_{ij} B_j}{Y} = \Phi\left(\epsilon_{\boldsymbol{s}}^b\right). \tag{3.11}$$

Proposition 3.4 reveals the intimate relationship between the existence condition and the construction of industrial bubbles, where the former is one of the leading eigenvectors of the matrix $\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^b\right) \mathcal{M}_{\boldsymbol{s}}$, and the latter depends on its leading eigenvalue. The condition (3.11) then selects the unique eigenvector for the industrial bubbles. In the bubble equilibrium, the efficiency cutoff is necessarily higher than in the bubbleless equilibrium, and the difference between ϵ^f and $\epsilon_{\boldsymbol{s}}^b$ identifies the size of the bubble required to finance investment demand for more efficient firms. The collateral value of the bubbles then equals the excess demand for credit (given the efficiency cutoff).

4. Bubble Contagion

Previous analysis has suggested that in financial linkages, industries are linked by investment efficiency. In this section, we examine a particular type of linkage — bubble contagion. By contagion, we mean that the existence of bubbles in one industry depends on the existence of bubbles in another industry or group of industries. Alternatively, the bursting of a bubble in one industry or group of industries also bursts bubbles in other industries. As emphasized in Biswas, Hanson, and Phan (2020) and Kocherlakota (2009), the bursting of asset bubbles can lead to large output losses and prolonged recessions, which are particularly painful in our model because they are contagious across industries.

We begin with the following corollary, which provides a mechanical way to test whether an economy exhibits bubble contagion.

Corollary 5 (Contagious Bubbles)**.** *Bubbles are contagious if there exist industry collections $\boldsymbol{s} \subset \boldsymbol{s}'$ such that the bubble equilibrium exists for $\boldsymbol{s}'$ but not for $\boldsymbol{s}$, that is,*

$$\rho\left[\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}'}^b\right) \mathcal{M}_{\boldsymbol{s}'}\right] > \frac{\chi}{\Gamma(\epsilon^f)} > \rho\left[\operatorname{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^b\right) \mathcal{M}_{\boldsymbol{s}}\right]. \tag{4.1}$$

That is, in a contagious bubble economy, some industries may be critical: the bursting of bubbles in these industries necessarily eliminates bubbles in some other industries, but not vice versa.

Definition 2. *A collection of industries $\boldsymbol{s}$ is critical if: (1) the $\boldsymbol{s}$-bubble equilibrium exists; (2) for any $\underline{\boldsymbol{s}} \subset \boldsymbol{s}$, no $\underline{\boldsymbol{s}}$-bubble equilibrium exists; (3) for some $\overline{\boldsymbol{s}} \supset \boldsymbol{s}$, $\overline{\boldsymbol{s}} \setminus \boldsymbol{s}$-bubble equilibrium does not exist.*

A bubble crash in a critical industry can drag down the entire stock market. Therefore, additional care should be taken when changes in financial conditions occur in a critical industry. For example, if the equity constraint is tightened in a non-critical industry, it may have a limited effect on the overall economy while bubbles still survive. On the other hand, if the equity constraint is tightened in a critical industry, bubbles in all industries may burst, leading to a significant decline in aggregate output.

The analysis of bubble contagion depends on the properties of the financial linkages $\mathcal{M}$. To illustrate the basic intuition, we provide a detailed characterization of the bubble contagion in a two-industry example, and then conduct a dynamic quantitative analysis in Section 5.

4.1 Illustration by a Two-Industry Model

Consider the possibility of bubble contagion in a two-industry model, where the 2×2 financial linkage matrix is denoted by

$$\boldsymbol{\mathcal{M}} = \begin{bmatrix} \mathcal{M}_{11} & 1-\mathcal{M}_{11} \\ 1-\mathcal{M}_{22} & \mathcal{M}_{22} \end{bmatrix}.$$

We also restrict our attention to the case with homogeneous capital and equity pledgeability, i.e., $\theta_j^b = \theta^b$ and $\theta_j^k = \theta^k$. This special case allows us to derive analytical results, and the intuition extends to more complicated models.

Proposition 4.1. *Bubbles in the two industries can coexist if and only if* $\phi^* \equiv \frac{\left(\frac{1}{\beta}-1\right)}{\Gamma\left(\epsilon^f\right)}\frac{1}{\theta^b} < 1$. *Furthermore, the following must hold:*

1 *(no contagion) If* $\mathcal{M}_{ii} > \phi^*$ *for* $i \in \{1,2\}$, *the bubbles are not contagious.*

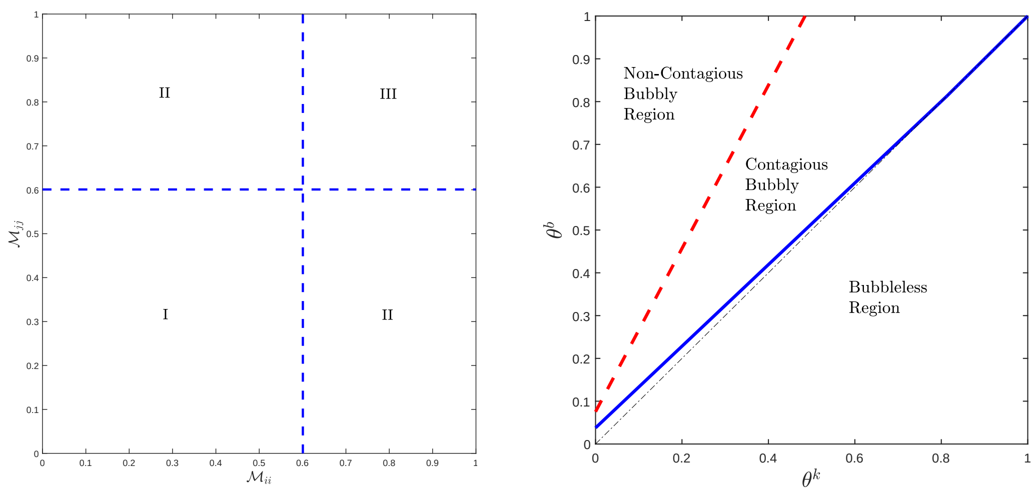
2 *(one-way contagion) If* $\mathcal{M}_{ii} < \phi^*$ *but* $\mathcal{M}_{jj} > \phi^*$, *then the burst of a bubble in industry* j *is contagious to industry* i, *but not vice versa.*

3 *(bidirectional contagion) If* $\mathcal{M}_{ii} < \phi^*$ *for all* $i \in \{1,2\}$, *then the bust of a bubble in one industry is contagious to the other and vice versa.*

Proposition 4.1 implies that the resilience of an industrial bubble to contagion increases with its self-reliance in the financial linkage matrix. This implication is similar to that in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Denbee et al. (2021): the importance of an agent or industry in the system depends on its linkage to other industries, in addition to its own size. As shown in the left panel of Figure 3, when $\mathcal{M}_{ii}$ is low, the collateral value of industry i depends more on the performance of industry j, and a bubble burst in industry j spillover to industry i. When $\mathcal{M}_{ii}$ is high, the bubble in industry i can survive regardless of the state of the other industries. Extrapolating the intuition to a more general environment, bubble contagion is more likely to occur when the off-diagonal elements are dense.

Note that in region II, if $\mathcal{M}_{ii} < \phi^*$ but $\mathcal{M}_{jj} > \phi^*$, then industry j is the critical industry. The bursting of the bubble in industry j eliminates the possibility of a bubble equilibrium, but the opposite is not true for industry i. Thus, the critical industry has a greater impact on the whole economy, which requires additional attention.

Figure 3: Bubbly Contagion



Proposition 4.1 divides the space $(\mathcal{M}_{ii}, \mathcal{M}_{jj})$ into three segments for a fixed combination of θ^b and θ^k. In the space of $\mathcal{M}_{ii}$ and $\mathcal{M}_{jj}$, the area of the infectious regions (II and III) is

$$C = 1 - (\phi^*)^2 .$$

If a bubble equilibrium exists, then for a fixed combination of $\mathcal{M}_{ii}$ and $\mathcal{M}_{jj}$, the larger C is, the more likely bubbles are contagious. Thus, C can be interpreted as the contagion risk. The following corollary connects the the contagion risk with the primitives of the economy.

Corollary 6. *Assuming $\phi^* < 1$, the contagion risk, $C = 1 - (\phi^*)^2$, increases with a smaller overall borrowing constraint θ^b, or a larger capital-specific pledgeability θ^k.*

The right panel of Figure 3 helps to visualize Corollary 6, where the parameter space is divided into three regions for a fixed financial linkage matrix. The intuition behind Corollary 6 is similar to that of the bubble equilibrium existence condition. With a lower θ^b, equity is less valuable as collateral, and the bubble existence condition in an industry becomes more stringent. With a higher θ^k, capital is more powerful in overcoming the financial frictions and the economy is less dependent on bubbles to provide liquidity; thus, industry bubbles are more fragile with respect to each other. The left panel illustrates how industrial interconnectedness leads to bubble contagion.[13]

[13]As an illustrative example, we assume that the distribution of investment efficiency is Pareto, $F(\epsilon) = 1 - (\epsilon/\underline{\epsilon})^{-\eta}$ with

5. Quantitative Study and Calibration

In this section, we exploit the theoretical properties of bubble equilibrium to shed light on the differential impact of an unexpected bubble burst in a critical versus a non-critical industry on aggregate outcomes. Importantly, we discipline the financial linkage matrix $\mathcal{M}$ based on U.S. merger and acquisition transaction data and examine the full transition dynamics following a bubble burst, which helps to provide a quantitative assessment of the importance of bubble contagion and an explanation of the stylized fact in Figure 1.

5.1 Extension with Production Network

To sharpen our understanding of the distinct roles of the production network and the financial network in propagating the business cycle, we first extend our baseline model to include input-output linkages. Our main findings on bubble contagion turn out to be robust to the introduction of the production network.

To do this, and to take into account the use of inputs from other industries, the production function for a firm in industry i is modified as follows:

$$o_{it}(\iota) = A_{it} k_{it}^{\alpha^k}(\iota)\, n_{it}^{\alpha^n}(\iota) \left(\prod_j x_{ijt}(\iota)^{\omega_{ij}} \right)^{\alpha^s}, \tag{5.1}$$

where A_{it} is the industry-specific TFP shock, and $x_{ijt}(\iota)$ the intermediate input from industry j to industry i. The parameters α^k, α^n, and α^s denote the capital, labor, and intermediate input shares, respectively. The input-output linkages are captured by the parameters ω_{ij}, which satisfy the restriction $\sum_j \omega_{ij} = 1$.

Denote the Domar weight of different industries as $\boldsymbol{\gamma} \equiv \{\gamma_1, \ldots, \gamma_S\}$, which is given by the standard Leontief inverse,

$$\boldsymbol{\gamma} = (\mathbf{I} - \mathrm{diag}(\alpha^s \boldsymbol{\omega}))^{-1} \boldsymbol{\varphi}.$$

In addition to determining the importance of the industrial TFP shock in GDP fluctuations, the Domar weight will also play an important role in shaping the properties of bubble equilibria in our environment.

Define the function $\Psi(x)$ as

$$\Psi(x) \equiv \sum_{j=1}^{S} \frac{\alpha^k \gamma_j}{\chi + \delta - \theta_j^k \Gamma(\varepsilon^f)} \left(\frac{\delta \varepsilon^f F(\varepsilon^f)}{\int_{\varepsilon > \varepsilon^f} \varepsilon dF} - \chi - \theta_j^k \right),$$

$\underline{\epsilon} = 1 - 1/\eta$ for normalization. We set $\eta = 4.2$, $M_{11} = M_{22} = 0.5$, $\beta = 0.99$, and $\delta = 0.04$.

which is the multi-industry counterpart of $\Phi(x)$ in equation (3.1). Note that the weight of different industries becomes $\alpha^k \gamma_j$, which reflects the impact of input-output linkages. The following proposition extends the theoretical results developed in section 3 to the production network environment.

Proposition 5.1. *With both production network and financial network, the following must hold:*

1. *In bubbleless steady state, the efficiency cutoff ε^f satisfies*

$$\Psi(\epsilon^f) = 0. \tag{5.2}$$

2. *For a selection of industries $\boldsymbol{s} \subset \mathcal{P}(\mathbf{S})$, there exists a unique* s*-bubble equilibrium if and only if*

$$\rho\left(diag\left(\boldsymbol{\theta}_s^b\right)\mathcal{M}_{\mathrm{s}}\right) > \frac{\chi}{\Gamma(\varepsilon^f)}. \tag{5.3}$$

In a $\boldsymbol{s}$-bubble steady state, the efficiency cutoff is

$$\varepsilon_s^b = \Gamma^{-1}\left(\frac{\chi}{diag\left(\boldsymbol{\theta}_s^b\right)\mathcal{M}_{\mathrm{s}}}\right),$$

and the vector of bubbles $\{B_j\}$ for $j \in \boldsymbol{s}$ is a leading eigenvector of the matrix $diag(\boldsymbol{\theta}_s^b)\mathcal{M}_{\mathrm{s}}$ such that

$$\frac{\sum_{j=\in \boldsymbol{s}}\sum_{i\in \boldsymbol{s}}\theta_j^b \mathcal{M}_{ji} B_i}{Y} = \Psi(\epsilon_s^b). \tag{5.4}$$

Starting from the bubbleless equilibrium, note that in the absence of intermediate inputs, the vector of Domar weights γ coincides with the final goods share φ. As a result, condition (5.2) nests the previous condition (3.2).

Next we turn to the bubble equilibrium. The existence of a bubble equilibrium depends on the production network only through its effect on the efficiency cutoff in the bubbleless equilibrium ε^f. With respect to the intensive margin, the relative size of the bubble across industries is independent of the production network, while the absolute size of the bubble depends on the production network only through the Domar weight. That is, the interaction between financial frictions and the input-output linkages in our model economy can be succinctly summarized by the function $\Psi(x)$, and the Domar weight serves as a sufficient statistic.

5.2 Calibration

Many of the parameters are standard in the literature. We choose a period of one year and set the discount rate χ to 4%. The capital share α^k and the intermediate input share α^s are set to 40% and

45%, respectively, corresponding to the average values in KLEMS. The depreciation rate δ is set to 10%, consistent with the average industrial depreciation rates obtained from the BEA for 2001.

Table 1: Targets and Associated Parameters

Parameter	Value & Target
Discount factor, χ	0.04
Capital share, α^k	0.22
Intermediate goods share, α^s	0.45
Depreciation rate, δ	0.1
Investment efficiency distribution, η	MPK in Bubbleless Steady State
Equity pledgeability, θ^b	Total Credit to non-financial Corporations
Capital pledgeability, θ^k	Investment Ratio
Final goods expenditure share, φ_j	OECD ICIO Tables
Production linkage, ω_{ij}	OECD ICIO Tables
Financial linkage, $\mathcal{M}_{ij}$	M&A SDC

We now turn to the parameters that are more specific to our model environment. We calibrate these parameters in the bubble steady state that admits the highest efficiency cutoff level.[14]

A crucial object in our exercise is the financial linkage matrix $\mathcal{M}$. To calibrate this matrix, we use the M&A database in the SDC, which contains data on merger and acquisition transactions between firms from 1999 to 2018. We interpret these transactions as indicators of the likelihood of a firm being sold from one industry to another. Operationally, we aggregate firms into three groups: financial, manufacturing, and services, which correspond to the industries in the model.

The mapping from the Thomson Financial Macro Industry Hierarchy code to the three industries is listed in Table A.2 in the Appendix. For each industry j, we compute the total transaction value with firms in industry j as targets, and compute the transaction value associated with acquiring firms in industry i. We then set the acquisition probability $\mathcal{M}_{ji}$ as the ratio of value acquired by industry i to the total transaction value from industry j. This results in the following acquisition probability matrix:

$$\boldsymbol{\mathcal{M}} = \begin{bmatrix} 0.959 & 0.021 & 0.020 \\ 0.172 & 0.787 & 0.041 \\ 0.173 & 0.059 & 0.767 \end{bmatrix},$$

where the order of the industries in $\mathcal{M}$ are finance, manufacturing, and service. An important feature is that the financial industry has the largest diagonal element and its probability of absorbing firms from the other two industries is the highest among the three. This feature makes the financial industry

[14] In our model environment, the bubble equilibrium is the stable equilibrium, and the bubbleless equilibrium is unstable in the sense that a local perturbation to the perceived bubble value in the bubbleless equilibrium will cause the economy to converge to the bubble equilibrium.

more likely to be a critical industry in determining the existence of bubbles.

The value added shares φ_j and the production network parameters ω_{ij} are obtained by aggregating the expenditure shares in 33 industries, obtained from the OECD Inter-Country Input-Output (ICIO) tables, into the aforementioned three industries mentioned above (with the same order of the three industries):[15]

$$\boldsymbol{\omega} = \begin{bmatrix} 0.187 & 0.172 & 0.641 \\ 0.018 & 0.634 & 0.348 \\ 0.089 & 0.249 & 0.662 \end{bmatrix}, \qquad \boldsymbol{\varphi} = [0.11, 0.29, 0.60] \, .$$

Note that the production network differs significantly from the financial linkage matrix: The financial industry is no longer the most critical industry in the production network. The mapping from the ICIO industry name to the model industry is shown in Table A.1 in the Appendix

In the baseline specification, we set the capital pledgeability and equity pledgeability to be the same across industries, that is, $\theta_j^k = \theta^k$ and $\theta_j^b = \theta^b$. We choose these two parameters so that the model matches the following two empirical moments: (1) the ratio of investment to output ($\frac{I}{Y}$) is 20%; (2) the ratio of total non-financial corporation credit to GDP is 80%, which is the average level in the U.S. around 2020 in the BIS data.

Finally, following Dong and Xu (2022), we assume that the investment efficiency shock follows a Pareto distribution, $F(\epsilon) = 1 - \left(\epsilon/\underline{\epsilon}\right)^{-\eta}$ with $\underline{\epsilon} = 1 - 1/\eta$ for normalization. We set the shape parameter η to pin down the marginal product of capital in the bubble steady state; namely, we set $\eta = 2.3$ so that R_j is 0.092, as in Caselli and Feyrer (2007).

5.3 Response to a Bubble Burst

In our calibration, there is a bubble equilibrium in which bubbles occur in all industries, and the economy is initially in this equilibrium. Our exercise examines the consequences of an unexpected bubble burst in one of the three industries, and we choose the equilibrium in which the bubbles continue to exist in the remaining industries whenever possible. That is, we look for the most favorable outcomes.

Table 2 reports the predictions of our calibrated model regarding the contagion properties of bubbles, where the order of industries (from left to right and from top to bottom) is finance first, manufacturing second, and services third. A cross sign in the (i, i) element means that a bubble bursts in industry i, and a cross sign in the (i, j) element means that a bubble bursting in industry i also causes a bubble to burst in industry j. On the other hand, a positive value in the (i, j) element indicates that a bubble bursting in industry i increases the size of the bubble in industry j rather than bursting it. The table clearly shows that when the financial bubble (first row and first column) bursts, the bubbles in the other two industries (in the first row) also burst. On the other hand, when the bubble in services or manufacturing bursts, the bubbles in the other industries remain intact—in fact, the bubbles in the

[15] We use information for the most recent year available, 2015.

other industries are slightly enlarged. Therefore, in our calibrated model economy, there is one-way bubble contagion and the financial industry is the critical industry.

This property is closely related to the pattern in the financial linkage matrix $\mathcal{M}$. As characterized in sections 3 and 4, the diagonal elements are responsible for whether an industrial bubble can survive on its own, and the off-diagonal elements determine which industries are critical for the existence of bubbles in other industries. In our calibrated economy, the bubble in the financial industry is self-sustaining because the acquisition probability $\mathcal{M}_{11}$ is large enough. The strong dependence of the bubbles in the other two industries on the financial industry is captured by the large off-diagonal elements $\mathcal{M}_{j1}$, $j = 2, 3$, which are an order of magnitude larger than other off-diagonal elements in the second and third columns. Note that the financial industry does not dominate the other two industries in terms of value added in the production network, illustrating the distinct role of the financial network in propagating the business cycle.

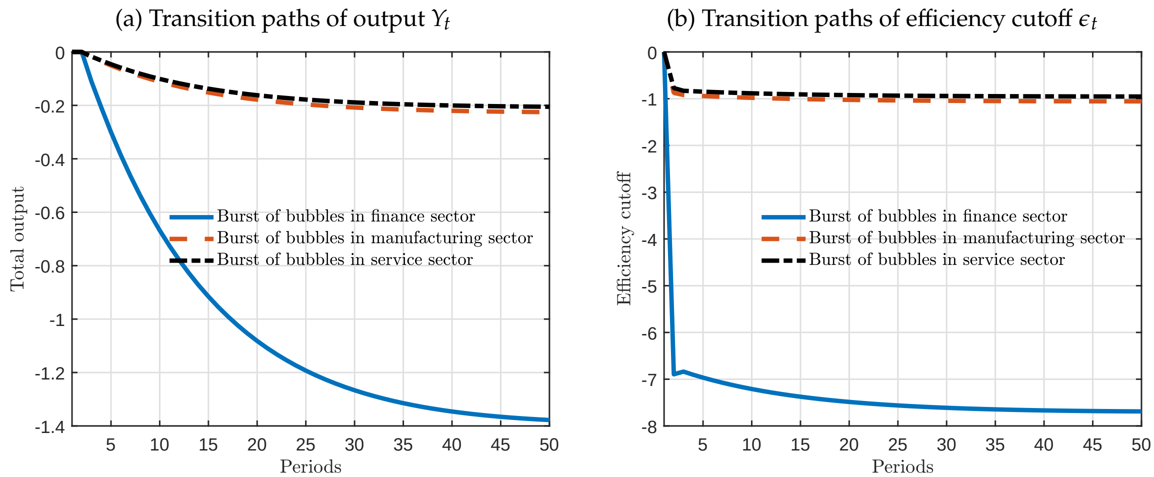
Table 2: Industrial Bubble Contagion

	FIN Bubble	MANU Bubble	SERVICE Bubble
FIN Burst	×	×	×
MANU Burst	+28.9%	×	+7.2%
SERVICE Burst	+25.4%	+12.6%	×

We now turn to the dynamics of the transition. The top row of panels in Figure 4 shows the dynamic responses of aggregate output (left panel) and the efficiency cutoff (right panel) to a bubble burst in one of the three industries: finance (solid blue lines), manufacturing (dashed red lines), and services (dashed black lines). First, the efficiency cutoff falls immediately after a bubble bursts in any one of the three industries (right-hand panel). This is because the cutoff is an increasing function of bubbles, whereas bubbles are purely forward-looking variables and their bursting therefore causes an immediate jump in the cutoff. However, comparing the three industrial bubbles, the decline in the efficiency cutoff is much larger when the financial bubble bursts than when the bubbles in the other two industries burst, precisely because the bursting of a financial bubble is contagious while the bursting of a bubble in the other two industries is not. The left-hand panel shows that the bursting of a bubble leads to gradual and permanent output losses, with the output loss from the bursting of a financial bubble being an order of magnitude larger than that from the bursting of other industrial bubbles. This is because after a bubble bursts, firms' borrowing constraints are tightened, forcing the relatively more efficient firms to cut back on investment and leading to lower aggregate investment efficiency. The lower investment efficiency leads to a lower capital stock and output level. As investment is the most affected by the bursting of a financial bubble, the corresponding output loss is also much larger.[16]

[16]By construction, both TFP and total labor supply are constant in the experiments, so the capital stock is the only factor

Figure 4: Output Loss and Efficiency Cutoff



To see whether the production network plays a role in bubble contagion across industries, Figure A.1 in Appendix C shows the dynamic effect of a bubble burst without production networks—that is, by setting $\alpha_j^s = 0$ or $x_{ijt}(\iota) = 1$ for all i and j in the production function (5.1). The pattern of bubble contagion, as well as the dynamics of output and the efficiency cutoff, remain the same as before, except that the magnitude of the recession is smaller in the steady state. This reconfirms our theoretical results that the direction of bubble contagion can be independent of the production network, although the production network helps to increase the size of the industrial bubbles.

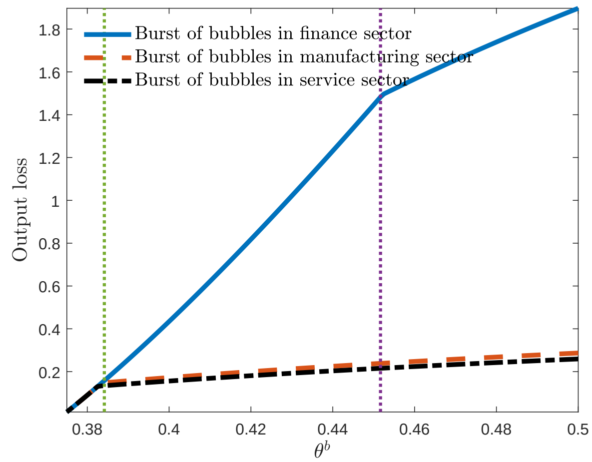
In summary, the severity of the recession depends on whether or not the triggering industry is a critical industry in the financial network, independent of the production network. This exercise thus illustrates the importance of the financial network, such that weak confidence in a critical industry within the financial linkages can lead to a series of cascading effects on other industries and drag the entire economy into a deep recession, as happened during the 2008 financial crisis shown in Figure 1. Macroprudential policies should therefore pay particular attention to critical industries in the financial network.

Role of Equity pledgeability θ^b. One of the key parameters in determining whether there is bubble contagion is the equity pledgeability θ^b. As illustrated in Corollary 2, a higher θ^b helps to sustain bubbles and reduce bubble contagion risk. Figure 5 displays how the steady-state output loss after the bubble burst in an industry depends on the level of equity pledgeability. Overall, the output loss increases in θ^b. This is because output is independent of θ^b when the economy enters the bubbleless

of production that can decline over time as investment efficiency deteriorates. Here we compute the transition paths of output Y_t and the efficiency cutoff ε_t after the burst of one of the three industrial bubbles, and all the values are percentage changes. The top two panels are computed using the baseline model with production network, while the bottom two panels are computed using the model without production network.

equilibrium, whereas output increases in θ^b in the bubble equilibrium.[17]

Figure 5: Output Loss Due to Burst of Different Industrial Bubbles



The two vertical dashed lines partition the figure into three regions. When θ^b is relatively small, the existence condition for bubbles is strict and the economy is in a two-way bubble contagion region. The bursting of any industrial bubble will eliminate the bubbles in other industries. Therefore, we see that the effects on total output are similar across industries. When θ^b takes an intermediate value, the economy is in the one-way bubble contagion region with finance as the critical industry. This is the region to which our baseline calibration belongs. In this region, the bursting of the bubble in the financial industry has a much larger impact on the economy than in the other two industries. Finally, when θ^b is relatively large, all industrial bubbles can survive independently and there is no more bubble contagion. As a result, the bursting of bubbles has a limited impact on output and the lines show lower slopes in this region.

5.4 Response to Exogenous Shocks

The endogenous bursting of bubbles due to a change in beliefs contributes to aggregate uncertainty. In this section, we show how the presence of bubbles modifies the economy's response to conventional business cycle shocks. It turns out that the answer to this question depends on the type of shocks considered. In the first experiment with shocks to the pledgeability of capital, bubbles help to stabilize the economy. In contrast, in the second experiment with shocks to the dispersion of investment technology, bubbles amplify the responses of aggregate variables.

Financial shock. In the initial steady state, the capital pledgeability is $\theta^k = 0.2$. In period 0, there is an unexpected shock that relaxes the financial constraint, increasing θ^k to $\theta^k = 0.25$. All agents believe

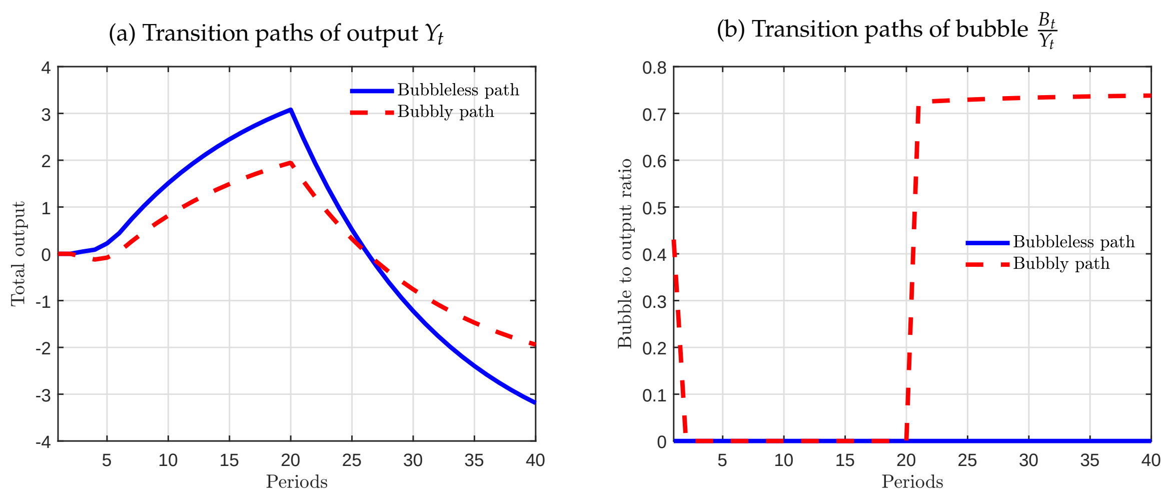
[17]In this figure, we change the value of θ^b and keep the other parameter values the same as in our baseline calibration, and calculate the output loss after a bubble burst in one of the three industries.

that this new level of financial constraint will last forever, until it unexpectedly drops to $\theta^k = 0.15$ in period 20.

The left panel of Figure 6 shows the responses of the aggregate output in this cycle for both bubbleless and bubble equilibria. In the bubbleless equilibrium, the allocation efficiency of investment goods increases with θ^k, since a higher level of θ^k allows firms with better investment technology to borrow more. It follows that there is a boom-bust cycle triggered by the change in the collateral constraint.

Interestingly, the magnitude of the boom and bust in the bubble equilibrium is smaller than that in the bubbleless equilibrium. With a higher level of θ^k, the need for bubble is dampened. With a sufficiently large increase of θ^k, bubbles can no longer be supported in the equilibrium. As the right panel of Figure 6 shows, during the boom, the bubbles in all industries burst, which helps to cool the economy. Conversely, when θ^k falls unexpectedly, the need for liquidity allows bubbles to form, which mitigates the recession. In this case, bubbles act as an automatic stabilizer in response to the financial shock.

Figure 6: Transition Paths with θ^k Cycle

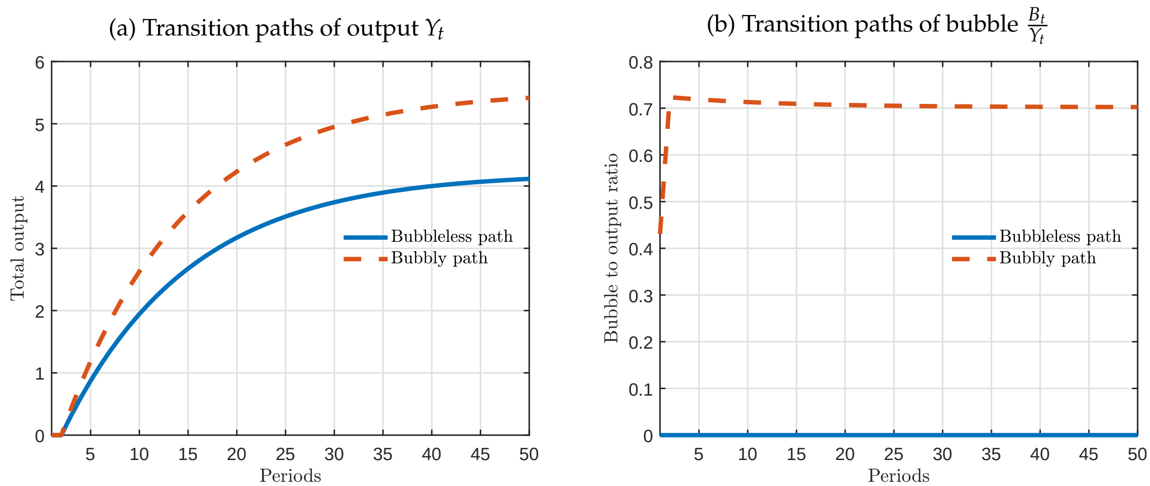


Investment Efficiency Dispersion Shock. Instead, consider a change in the distribution of the idiosyncratic investment efficiency ε. Recall that this distribution follows a Pareto distribution with shape parameter η. Suppose the economy experiences an unexpected decline in η from 2.3 to 2.1, so that the distribution of ε is more dispersed and the need for reallocation is stronger.

In the bubbleless equilibrium, the economy still benefits from the increased dispersion because there are better investment opportunities, although its ability to take advantage of these opportunities is limited by financial frictions. In the bubble equilibrium, the need for liquidity creates additional space for bubbles. Accordingly, the borrowing capacity of firms is increased and the economy can better exploit the increased dispersion. As shown in Figure 7, aggregate output increases more in the bubble equilibrium, accompanied by an increase in the bubble-to-output ratio.

These two examples illustrate that bubbles can act as both stabilizers and accelerators. Depending on whether the underlying shock requires additional liquidity and in which direction it pushes the bubbleless economy, the presence of bubbles can dampen or amplify its effects.

Figure 7: Transition Paths with Shock to Dispersion



6. Conclusion

Sometimes the bursting of a bubble in one industry (such as the bursting of the IT bubble in 2000) does not have a serious impact on the stock prices of other industries or on overall investment activity in the economy. At other times, however, the bursting of a bubble in a critical industry (such as the bursting of the housing bubble in 2007) has a huge impact on stock prices in other industries and on overall investment activity in the economy.

This paper proposes a framework for determining what constitutes a critical industry and studies contagious stock price bubbles in a multi-industry economy. In our model, stock price bubbles arise endogenously to overcome financial frictions and help inject additional liquidity into the credit system. Due to cross-industry financial linkages through collateral constraints and mergers and acquisitions, the existence of bubbles in different industries may be interdependent. We characterize the full set of bubble equilibria and provide the condition under which bubbles are unidirectionally contagious, i.e., the bursting of a bubble in one industry leads to the bursting of bubbles in other industries, but not vice versa. Quantitatively, we calibrate the financial linkages in our model using U.S. merger and acquisition data and show that an unexpected burst of a bubble in the financial industry can indeed have a contagious effect and lead to a severe recession, while the burst of a bubble in the manufacturing industry does not have a contagious effect on bubbles in other industries and leads to a much less severe recession.

References

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2015. "Systemic risk and stability in financial networks." *American Economic Review* 105 (2):564–608.

Arce, Óscar and David López-Salido. 2011. "Housing bubbles." *American Economic Journal: Macroeconomics* 3 (1):212–41.

Asriyan, Vladimir, Luca Fornaro, Alberto Martin, and Jaume Ventura. 2016. "Monetary policy for a bubbly world." Tech. rep., National Bureau of Economic Research.

Bellman, Richard. 1997. *Introduction to matrix analysis*. SIAM.

Bengui, Julien and Toan Phan. 2018. "Asset pledgeability and endogenously leveraged bubbles." *Journal of Economic Theory* .

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." *Handbook of macroeconomics* 1:1341–1393.

Biswas, Siddhartha, Andrew Hanson, and Toan Phan. 2020. "Bubbly recessions." *American Economic Journal: Macroeconomics* 12 (4):33–70.

Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo. 2017. "Risk sharing and contagion in networks." *The Review of Financial Studies* 30 (9):3086–3127.

Caselli, Francesco and James Feyrer. 2007. "The marginal product of capital." *The quarterly journal of economics* 122 (2):535–568.

Denbee, Edward, Christian Julliard, Ye Li, and Kathy Yuan. 2021. "Network risk and key players: A structural analysis of interbank liquidity." *Journal of Financial Economics* 141 (3):831–859.

Dong, Feng, Jianjun Miao, and Pengfei Wang. 2020. "Asset bubbles and monetary policy." *Review of Economic Dynamics* .

Dong, Feng and Zhiwei Xu. 2022. "Bubbly bailout." *Journal of Economic Theory* 202:105460.

Farhi, Emmanuel and Jean Tirole. 2011. "Bubbly liquidity." *The Review of Economic Studies* 79 (2):678–706.

Gali, Jordi. 1994. "Government size and macroeconomic stability." *European Economic Review* 38 (1):117–132.

Galí, Jordi. 2014. "Monetary policy and rational asset price bubbles." *American Economic Review* 104 (3):721–52.

Gomes, Joao, Urban Jermann, and Lukas Schmid. 2016. "Sticky leverage." *American Economic Review* 106 (12):3800–3828.

Hirano, Tomohiro, Masaru Inaba, and Noriyuki Yanagawa. 2015. "Asset bubbles and bailouts." *Journal of Monetary Economics* 76:S71–S89.

Hirano, Tomohiro and Noriyuki Yanagawa. 2016. "Asset bubbles, endogenous growth, and financial frictions." *The Review of Economic Studies* 84 (1):406–443.

Huremovic, Kenan, Gabriel Jiménez, Enrique Moral-Benito, Fernando Vega-Redondo, José-Luis Peydró et al. 2020. "Production and financial networks in interplay: Crisis evidence from supplier-customer and credit registers." .

Ikeda, Daisuke and Toan Phan. 2016. "Toxic asset bubbles." *Economic Theory* 61 (2):241–271.

———. 2019. "Asset bubbles and global imbalances." *American Economic Journal: Macroeconomics* 11 (3):209–51.

Jackson, Matthew O and Agathe Pernoud. 2021. "Systemic risk in financial networks: A survey." *Annual Review of Economics* 13:171–202.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic effects of financial shocks." *American Economic Review* 102 (1):238–271.

Kiyotaki, Nobuhiro and John Moore. 1997. "Credit cycles." *Journal of Political Economy* 105 (2):211–248.

Kocherlakota, Narayana. 2009. "Bursting bubbles: Consequences and cures." *Unpublished manuscript, Federal Reserve Bank of Minneapolis* 84.

Kopytov, Alexandr. 2018. "Booms, busts, and common risk exposures." *Available at SSRN 3290616* .

Martin, Alberto and Jaume Ventura. 2012. "Economic growth with bubbles." *American Economic Review* 102 (6):3033–58.

———. 2016. "Managing credit bubbles." *Journal of the European Economic Association* 14 (3):753–789.

———. 2018. "The macroeconomics of rational bubbles: a user's guide." *Annual Review of Economics* 10:505–539.

McKay, Alisdair and Ricardo Reis. 2016. "The role of automatic stabilizers in the US business cycle." *Econometrica* 84 (1):141–194.

Miao, Jianjun. 2014. “Introduction to economic theory of bubbles.” *Journal of Mathematical Economics* 53:130–136.

Miao, Jianjun and Pengfei Wang. 2014. “Sectoral bubbles, misallocation, and endogenous growth.” *Journal of Mathematical Economics* 53:153–163.

———. 2018. “Asset Bubbles and Credit Constraints.” *American Economic Review* .

Miao, Jianjun, Pengfei Wang, and Zhiwei Xu. 2015. “A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles.” *Quantitative Economics* 6 (3):599–635.

Moll, Benjamin. 2014. “Productivity losses from financial frictions: Can self-financing undo capital misallocation?” *American Economic Review* 104 (10):3186–3221.

Noutsos, Dimitrios. 2008. “Perron Frobenius theory and some extensions.” *Department of Mathematics, University of Ionnina, Greece* .

Wang, Pengfei and Yi Wen. 2012a. “Hayashi meets Kiyotaki and Moore: A theory of capital adjustment costs.” *Review of Economic Dynamics* 15 (2):207–225.

———. 2012b. “Speculative bubbles and financial crises.” *American Economic Journal: Macroeconomics* 4 (3):184–221.

Appendix

A. Data Appendix

In this section we first show the mapping from the industry names in the ICIO to the three industries in our model to calibrate the production network:

Table A.1: Correspondence of Model Industry to Real Industry

Model Industry	Industry Name in ICIO
Finance Industry	Real Estate
Manufacturing Industry	Food, beverages, tobacco; Textile; Wood; Paper; Coke,petroleum; Chemicals; Rubber and plastics; Other mineral; Basic metals; Machinery, nec; Electrical and opt. equipment; Transport equipment; Manufacturing, nec; Motor vehicles; Agriculture; Mining and quarrying; Construction
Service Industry	Wholesale trade; Retail trade; Transport and storage; Post and telecom.; Electricity, gas, water; Hotel, restauration; Public admin.; Education; Health, social work; Other comm. services; Renting of m&eq.; Financial intermediation

We then show the mapping from the Thomson Financial Macro Industry Hierarchy to the three industries in our model to calibrate the financial network matrix $\mathcal{M}$:

Table A.2: Correspondence of Model Industry to Real Industry

Model Industry	Industry Name in Macro Code
Finance Industry	Finance
Manufacturing Industry	Consumer Products and Services, Energy and Power, High Technology, Industrials, Materials, Consumer Staples
Service Industry	Real estate, Retail, Healthcare, Media and Entertainment, Government and Agencies, Telecommunications

B. Proofs

Proof of Proposition 2.1 and Corollary 1. Since production functions have constant returns to scale at the firm level, we guess and verify that the value function of firm ι in industry j adopts a linear form:

$$V_{jt}\left(k_{jt}(\iota), l_{jt}(\iota), \epsilon_{jt}(\iota)\right) = v^k_{jt}\left(\epsilon_{jt}(\iota)\right) k_{jt}(\iota) - v^l_{jt}\left(\epsilon_{jt}(\iota)\right) l_{jt}(\iota) + b_{jt}\left(\epsilon_{jt}(\iota)\right), \tag{B.1}$$

where the additional term $b_{jt}\left(\epsilon_{jt}(\iota)\right) \geq 0$ is the stock bubble in industry j. Using (B.1), we can rewrite the recursive problem for firm-ι in industry j as follows:

$$V_{jt}\left(k_{jt}(\iota), l_{jt}(\iota), \epsilon_{jt}(\iota)\right) = \max_{i_{jt}(\iota), l_{j,t+1}(\iota)} \left\{ d_{jt}(\iota) + \mathbb{E}\frac{\beta\Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(k_{j,t+1}(\iota), l_{j,t+1}(\iota), \epsilon_{j,t+1}(\iota)\right) \right\} \tag{B.2}$$

$$= \max_{i_{jt}(\iota), l_{j,t+1}(\iota)} \left\{ R_{jt} k_{jt}(\iota) - i_{jt}(\iota) - l_{jt}(\iota) + l_{j,t+1}(\iota)/R_{ft} \right. \tag{B.3}$$

$$\left. + Q_{jt}\left[(1-\delta) k_{jt}(\iota) + \epsilon_{jt}(\iota) i_{jt}(\iota)\right] + B_{jt} - Q_{jt}^{L} l_{j,t+1}(\iota) \right\}, \tag{B.4}$$

subject to

$$0 \leq i_{jt}(\iota) \leq R_t k_{jt}(\iota) - l_{jt}(\iota) + l_{j,t+1}(\iota)/(1+r_t).$$

Define

$$Q_{jt} \equiv \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} v_{j,t+1}^{k}\left(\epsilon_{j,t+1}(\iota)\right), \tag{B.5}$$

$$B_{jt} \equiv \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} b_{j,t+1}\left(\epsilon_{j,t+1}(\iota)\right),$$

$$Q_{jt}^{L} \equiv \mathbb{E}_t \frac{\beta\Lambda_{t+1}}{\Lambda_t} v_{j,t+1}^{l}\left(\epsilon_{j,t+1}(\iota)\right). \tag{B.6}$$

Also define

$$\theta_j^k \equiv \xi_j \sigma_j,\ \theta_j^b \equiv \xi_j,\ \overline{Q}_{jt} \equiv \sum_{i \in \mathbf{S}} \mathcal{M}_{ji} Q_{it},\ \overline{B}_{jt} \equiv \sum_{i \in \mathbf{S}} \mathcal{M}_{ji} B_{it}.$$

Then the borrowing constraint (2.11) can be rewritten as

$$l_{j,t+1}(\iota)/(1+r_t) \leq \xi_j \left(\sigma_j \overline{Q}_{jt} k_{jt}(\iota) + \overline{B}_{jt}\right) = \theta_j^k \overline{Q}_{jt} k_{jt}(\iota) + \theta_j^b \overline{B}_{jt}, \tag{B.7}$$

Note that (B.4) is a linear function of $i_{jt}(\iota)$ and $l_{j,t+1}(\iota)$. The coefficient before $i_{jt}(\iota)$ is $Q_{jt}\epsilon_{jt}(\iota) - 1$, and the coefficient before $l_{j,t+1}(\iota)$ is $1/(1+r_t) - Q_{jt}^{l}$. We guess and verify that the firm's decision rules follow a cut-off strategy, so there are two possible cases to consider:

Case A: $\epsilon_{jt}(\iota) < \epsilon_{jt}^{*} \equiv 1/Q_{jt}$. In this case, any firm ι in industry j with an investment efficiency shock below the industry cutoff ϵ_{jt}^{*} will not invest: $i_{jt}(\iota) = 0$. We assume that firms' dividends are positive in this case, so optimizing over $l_{j,t+1}(\iota)$ yields $Q_{jt}^{L} = 1/(1+r_t)$. Since the aggregate level of debt L_{jt} in each industry can be either positive or negative in equilibrium, a firm's debt position $l_{j,t+1}(\iota)$ is indeterminate under Case A. The optimal condition for investment implies that the cutoff is equal to the inverse of Tobin's Q: $\epsilon_{jt}^{*} \equiv 1/Q_{jt}$.

Case B: $\epsilon_{jt}(\iota) \geq \epsilon_{jt}^{*}$. In this case, it is optimal to invest by borrowing as much as possible, so the dividend is zero and the borrowing constraint (B.7) is binding:

$$l_{j,t+1}(\iota)/(1+r_t) = \theta_j^k \overline{Q}_{jt} k_{jt}(\iota) + \theta_j^b \overline{B}_{jt}. \tag{B.8}$$

Therefore, we have the following decision rules:

$$i_{jt}(\iota) = \begin{cases} R_{jt}k_{jt}(\iota) + \theta_j^k \overline{Q}_{jt} k_{jt}(\iota) + \theta_j^b \overline{B}_{jt} - l_{jt}(\iota), & \text{if } \epsilon_{jt}(\iota) \geq \epsilon_{jt}^* \\ 0, & \text{if } \epsilon_{jt}(\iota) < \epsilon_{jt}^* \end{cases}. \tag{B.9}$$

$$d_{jt}(\iota) = \begin{cases} 0, & \text{if } \epsilon_{jt}(\iota) \geq \epsilon_{jt}^* \\ \text{indeterminate}, & \text{if } \epsilon_{jt}(\iota) < \epsilon_{jt}^* \end{cases}. \tag{B.10}$$

$$l_{j,t+1}(\iota)/(1+r_t) = \begin{cases} \theta_j^k \overline{Q}_{jt} k_{jt}(\iota) + \theta_j^b \overline{B}_{jt}, & \text{if } \epsilon_{jt}(\iota) \geq \epsilon_{jt}^* \\ \text{indeterminate}, & \text{if } \epsilon_{jt}(\iota) < \epsilon_{jt}^* \end{cases}. \tag{B.11}$$

Substituting the decision rule for investment and the equation $Q_{jt}^L = 1/(1+r_t)$ into (B.2) yields

$$\begin{aligned} V_{jt}\left(k_{jt}(\iota), l_{jt}(\iota), \epsilon_{jt}(\iota)\right) \;&=\; R_{jt}k_{jt}(\iota) + Q_{jt}(1-\delta)k_{jt}(\iota) + B_{jt} - l_{jt}(\iota) \\ &\quad + \max\left(\epsilon_{jt}(\iota)/\epsilon_{jt}^* - 1, 0\right) \cdot \left(R_{jt}k_{jt}(\iota) - l_{jt}(\iota) + l_{j,t+1}(\iota)/(1+r_t)\right), \end{aligned} \tag{B.12}$$

in which $l_{j,t+1}(\iota)/(1+r_t)$ is given by equation (B.8). Then matching the coefficients on both sides of (B.12) under the conjecture in (B.1) gives

$$\begin{aligned} v_{jt}^k\left(\epsilon_{jt}(\iota)\right) &= R_{jt} + (1-\delta)Q_{jt} + \max\left(\epsilon_{jt}(\iota)/\epsilon_{jt}^* - 1, 0\right) \cdot \left(R_{jt} + \theta_j^k \overline{Q}_{jt}\right), \\ b_{jt}\left(\epsilon_{jt}(\iota)\right) &= B_{jt} + \theta_j^b \max\left(\epsilon_{jt}(\iota)/\epsilon_{jt}^* - 1, 0\right) \overline{B}_{jt}, \\ v_{jt}^l\left(\epsilon_{jt}(\iota)\right) &= 1 + \max\left(\epsilon_{jt}(\iota)/\epsilon_{jt}^* - 1, 0\right). \end{aligned}$$

Applying the definitions of Q_{jt}, B_{jt}, B_t, and Q_{jt}^L in (B.5) - (B.6) we get

$$\begin{aligned} Q_{jt} &= \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{j,t+1}\left(1 + \Gamma_{j,t+1}\right) + (1-\delta)Q_{j,t+1} + \theta_j^k \Gamma_{j,t+1} \overline{Q}_{j,t+1}\right], \\ B_{jt} &= \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(B_{j,t+1} + \theta_j^b \Gamma_{j,t+1} \overline{B}_{j,t+1}\right), \\ \frac{1}{1+r_t} &= \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 + \Gamma_{j,t+1}\right), \end{aligned}$$

where

$$\Gamma_{jt} \equiv \mathbb{E} \max\left(\epsilon_j/\epsilon_{jt}^* - 1, 0\right) = \int_{\epsilon_{jt}^*}^{\bar{\epsilon}_j} \left(\epsilon_j/\epsilon_{jt}^* - 1\right) dF_j.$$

Moreover, integrating the individual investment decision using (B.9) immediately yields

$$I_{jt} \equiv \int_0^1 i_{jt}(\iota)\, d\iota = \left[R_{jt}K_{jt} + \theta_j^k \overline{Q}_{jt} K_{jt} + \theta_j^b \overline{B}_{jt} - L_{jt}\right] \cdot \left[1 - F\left(\epsilon_{jt}^*\right)\right]. \tag{B.13}$$

Similarly, integrating the individual law of motion for capital in industry j gives

$$K_{j,t+1} = (1-\delta) K_{jt} + \int_0^1 \int_{\underline{\epsilon}_j}^{\overline{\epsilon}_j} \epsilon_{jt}(\iota)\, i_{jt}(\iota)\, dF_j\left(\epsilon_{jt}(\iota)\right) d\iota = (1-\delta) K_{jt} + \mathbb{E}_j\left(\epsilon_j | \epsilon_j \geq \epsilon_{jt}^*\right) I_{jt}.$$

Proof of Proposition 3.1. From equation (2.17) and (2.18) in the bubbleless steady state (with $B_j = 0$ for all j) we get:

$$I_j = \left[R_j K_j + \theta_j^k Q^f K_j - L_j\right] \cdot \left[1 - F(\varepsilon^f)\right],$$

and

$$I_j = \frac{\delta K_j}{\mathbb{E}(\varepsilon | \varepsilon \geq \varepsilon^f)}.$$

Note that here we use the fact that $Q_j = Q^f$ in the bubbleless steady state and that $\sum_{i=1}^S \mathcal{M}_{ji} = 1$. Combining the two we get the expression for L_j:

$$L_j = R_j K_j + \theta_j^k Q^f K_j - \frac{I_j}{1 - F(\varepsilon^f)} = R_j K_j + \theta_j^k Q^f K_j - \frac{\delta K_j}{\int_{\varepsilon > \varepsilon^f} \varepsilon dF(\varepsilon)}.$$

Divide both sides by the total output Y and use the facts $R_j K_j = \alpha \varphi_j Y$ and the expression for Q^f in the steady state:

$$Q^f = \frac{R_j\left(1 + \Gamma(\varepsilon^f)\right)}{\chi + \delta - \theta_j^k \Gamma(\varepsilon^f)},$$

we get that the universal cutoff is determined by

$$\Phi(\epsilon^f) = 0,$$

where $\chi = \frac{1}{\beta} - 1$, and $\Phi(x)$ is given by

$$\Phi(x) = \sum_{j=1}^S \frac{\alpha \varphi_j}{\chi + \delta - \theta_j^k \Gamma(x)} \left(\frac{\delta x F(x)}{\int_{\varepsilon > x} \varepsilon dF(\varepsilon)} - \chi - \theta_j^k\right).$$

Proof of Proposition 3.2. Equation (3.4) can be written in a more compact way as

$$\left[\frac{\chi}{\Gamma(\epsilon^*)} \mathbf{I} - \mathrm{diag}(\boldsymbol{\theta}_{\boldsymbol{s}}^b) \mathcal{M}_{\boldsymbol{s}}\right] \mathbf{B}_{\boldsymbol{s}} = 0, \tag{B.14}$$

where $\chi = \frac{1}{\beta} - 1$. Then, using the Perron-Frobenius theorem, it follows that there is a unique $\boldsymbol{s}$-bubble equilibrium if and only if

$$\rho\left(\mathrm{diag}\left(\boldsymbol{\theta}_{\boldsymbol{s}}^b\right) \mathcal{M}_{\boldsymbol{s}}\right) > \frac{\chi}{\Gamma(\epsilon^f)}.$$

Moreover, according to Assumption 1, since $\Phi(x)$ strictly increases with x, combining equations

(3.2) and (3.6) implies that

$$\epsilon_{\boldsymbol{s}}^{b} = \Gamma^{-1}\left(\frac{\chi}{\rho\left(\mathrm{diag}(\boldsymbol{\theta}_{\boldsymbol{s}}^{b})\mathcal{M}_{\boldsymbol{s}}\right)}\right) > \epsilon_{f}.$$

Proof of Corollary 3, Corollary 2, and Corollary 4. Based on equation (B.14), applying the results in Bellman (1997) and Noutsos (2008), one obtains Corollary 3, Corollary 2, and Corollary 4.

Proof of Proposition 3.3. In steady state, equation (2.12) implies that $\epsilon_j^* = \epsilon^*$ for all j, where the common cutoff ϵ^* is characterized by

$$\Gamma = \Gamma\left(\epsilon^*\right) = \int_{\epsilon^*}^{\bar{\epsilon}} \left(\frac{\epsilon}{\epsilon^*} - 1\right) dF\left(\epsilon\right) = \frac{1}{\beta R_f} - 1.$$

In turn, $Q_j = Q = \overline{Q}_j = 1/\epsilon^*$. Moreover, the Euler equation of Q_{jt} in (2.14) reveals that the industrial MPK is given by

$$R_j = \frac{\chi + \delta - \theta_j^k \Gamma}{1+\Gamma} Q, \tag{B.15}$$

where $\chi = \frac{1}{\beta} - 1$. Meanwhile, we can prove that

$$Y = \overline{AK}^{\alpha} N^{1-\alpha}. \tag{B.16}$$

where

$$\begin{aligned}
\overline{A} &\equiv \prod_{j\in\mathbf{S}} \lambda_j^{\varphi_j \alpha_j^l} \cdot \prod_{j\in\mathbf{S}} A_j^{\varphi_j}, \\
\overline{K} &\equiv \left[\prod_{j=1}^{S} K_j^{\varphi_j \alpha_j^k}\right]^{\frac{1}{\alpha}}.
\end{aligned}$$

and $\alpha = \boldsymbol{\varphi}'\boldsymbol{\alpha}^k$, $\lambda_j \equiv \frac{\varphi_j \alpha_j^l}{\sum_{i\in\mathbf{S}} \varphi_i \alpha_i^l}$.

We can easily verify that,

$$K_j = \frac{\varphi_j \alpha_j^k}{R_j} Y, \tag{B.17}$$

and

$$\kappa_j = \frac{\varphi_j \alpha_j^k / R_j}{\sum_{i\in\mathbf{S}}^{k} \varphi_i \alpha_i^k / R_i},$$

where R_j is the MPK of industry j, as characterized in (B.15).

Equation (B.13) implies

$$I_j = \frac{\delta_j}{\mathbb{E}_j\left(\epsilon_j | \epsilon_j \geq \epsilon_j^*\right)} K_j = \frac{\delta_j^k \varphi_j \alpha_j^k}{\mathbb{E}_j\left(\epsilon_j | \epsilon_j \geq \epsilon_j^*\right) R_j} Y. \tag{B.18}$$

Given N, substituting equation (B.17) into (B.16) yields

$$Y = \overline{A}\left[\prod_{j\in\mathbf{S}}\left(\frac{\varphi_j \alpha_j^k}{R_j}\right)^{\varphi_j \alpha_j^k}\right] Y^{\alpha} N^{1-\alpha}.$$

Since we have normalized $N = 1$, we immediately have

$$Y = \left\{\overline{A}\left[\prod_{j\in\mathbf{S}}\left(\frac{\varphi_j \alpha_j^k}{R_j}\right)^{\varphi_j \alpha_j^k}\right]\right\}^{\frac{1}{1-\alpha}}, \tag{B.19}$$

Then taking log on equation (B.19) yields

$$\log Y = \frac{\alpha}{1-\alpha}\left\{\log \epsilon^* + \log(1+\Gamma(\epsilon^*)) - \sum_{j=1}^{S} \varphi_j \log\left(\chi + \delta - \theta_j^k \Gamma(\epsilon^*)\right)\right\} + \texttt{constant}.$$

Proof of Proposition 3.4. Combining equations (2.17), (2.18), and (B.14) yields equation (3.11), i.e.,

$$\frac{\sum_{j\in\boldsymbol{s}} \theta_j^b \mathcal{M}_{ij} B_j}{Y} = \Phi\left(\epsilon_{\boldsymbol{s}}^b\right).$$

Proof of Corollary 5. We can obtain Corollary 5 by directly applying Proposition 3.2.

Proof of Proposition 4.1. We can obtain Proposition 4.1 by applying Corollary 5 with $\theta_j^b = \theta^b$ and $\theta_j^k = \theta^k$ in the two-industry case.

Proof of Corollary 6. As shown in Proposition 4.1, $\phi^* \equiv \frac{\chi}{\Gamma(\epsilon^f)}\frac{1}{\theta^b}$, where $\chi = \frac{1}{\beta} - 1$. Then we know that the contagion risk $C = 1 - \left(\phi^*\right)^2$ increases with a smaller overall borrowing constraint θ^b, or a larger capital-specific pledgeability θ^k.

Proof of Proposition 5.1. For any firm $\iota \in [0,1]$ in industry $i \in \mathbf{S}$, the FOCs for $\left(n_{it}(\iota), s_{ijt}(\iota)\right)$ are given, respectively, by

$$W_t = \alpha_i^l \frac{P_{it} o_{it}(\iota)}{n_{it}(\iota)} = \alpha_i^l \frac{P_{it} O_{it}}{N_{it}}, \tag{B.20}$$

$$P_{jt} = \alpha_i^s \omega_{ij} \frac{P_{it} o_{it}(\iota)}{s_{ijt}(\iota)} = \alpha_i^s \omega_{ij} \frac{P_{it} O_{it}}{S_{ijt}}, \tag{B.21}$$

Given $O_{it} \equiv \int_0^1 o_{it}(\iota)\, d\iota$ and similar definitions for L_{it}, S_{ijt}, and K_{it}, equation (2.5) implies that

$$R_{it} = \alpha_i^k \frac{P_{it} o_{it}(\iota)}{k_{it}(\iota)} = \alpha_i^k \frac{P_{it} O_{it}}{K_{it}}. \tag{B.22}$$

The resource constraint in industry j is given by

$$O_{jt} = X_{jt} + \sum_{i \in \mathbf{S}} S_{ijt}, \tag{B.23}$$

which can be rewritten as

$$\frac{P_{jt} O_{jt}}{Y_t} = \frac{P_{jt} X_{jt}}{Y_t} + \sum_{i \in \mathbf{S}} \frac{P_{jt} S_{ijt}}{Y_t}. \tag{B.24}$$

To recap, we have shown in equation (2.3) that

$$\frac{P_{jt} X_{jt}}{Y_t} = \varphi_j. \tag{B.25}$$

Denote $\gamma_{jt} = P_{jt} O_{jt} / Y_t$. Then the resource constraint (B.24) becomes

$$\gamma_{jt} = \varphi_j + \sum_{i \in \mathbf{S}} \alpha_i^s \omega_{ij} \gamma_{it}.$$

Then we have

$$\boldsymbol{\gamma} = \left(\mathbf{E} - \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}'\right)^{-1} \boldsymbol{\varphi},$$

and thus γ_{jt} is constant and is denoted as γ_j, where $\mathbf{E}$ denotes the $S \times 1$ unity matrix, $\boldsymbol{\gamma}$ denotes $S \times 1$ modified Dommar weight. After solving $\boldsymbol{\gamma}$, we know that, for all t, the expenditure ratio of industry j is given by

$$\frac{P_{jt} O_{jt}}{Y_t} = \gamma_j. \tag{B.26}$$

The demand for $\left(N_{it}, S_{ijt}, K_{it}\right)$ in (B.20), (B.21), and (B.22) can be further written as

$$N_{it} = \alpha_i^l \gamma_i \frac{Y_t}{W_t}, \tag{B.27}$$

$$S_{ijt} = \alpha_i^s \omega_{ij} \frac{\gamma_i}{\gamma_j} O_{jt}, \tag{B.28}$$

$$K_{it} = \alpha_i^k \gamma_i \frac{Y_t}{R_{it}}. \tag{B.29}$$

Note that here we get $R_j K_j = \alpha_j^k \gamma_k Y$ in the steady state, then follow the same process in the proof of proposition 3.1 we can get the expression of ε^f.

Then the market-clearing condition in the labor market is given by

$$N_t = \sum_{i \in \mathbf{S}} N_{it} = \boldsymbol{\gamma}' \boldsymbol{\alpha}^l \frac{Y_t}{W_t},$$

which implies

$$W_t = \boldsymbol{\gamma}' \boldsymbol{\alpha}^l \frac{Y_t}{N_t}. \tag{B.30}$$

In turn, equation (B.27) can be rewritten as

$$N_{it} = \lambda_i N_t,$$

where $\lambda_i = \frac{\gamma_i \alpha_i^l}{\boldsymbol{\gamma}' \boldsymbol{\alpha}^l}$.

Now we characterize the aggregation of Y_t. The industrial technology in equation (2.4) implies that

$$\ln O_{it} = \ln A_{it} + \alpha_i^k \ln K_{it} + \alpha_i^l \ln N_{it} + \sum_{j \in \mathbf{S}} \alpha_i^s \omega_{ij} S_{ijt}. \tag{B.31}$$

Substituting equation (B.28) into (B.31) with some algebraic manipulation yields

$$\ln O_{it} = \alpha_i^s \ln o_i + \ln A_{it} + \alpha_i^k \ln K_{it} + \alpha_i^l \ln N_{it} + \sum_{j \in \mathbf{S}} \alpha_i^s \omega_{ij} \ln O_{jt}, \tag{B.32}$$

where

$$\ln o_i \equiv \sum_{j \in \mathbf{S}} \ln \left(\alpha_i^s \omega_{ij} \gamma_i / \gamma_j \right)^{\omega_{ij}}.$$

Writing equation (B.32) in a more compact way yields

$$\ln \mathbf{O}_t = \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \ln \mathbf{o} + \ln \mathbf{A}_t + \mathbf{Diag}\left(\boldsymbol{\alpha}^k\right) \ln \mathbf{K}_t + \mathbf{Diag}\left(\boldsymbol{\alpha}^l\right) \ln \mathbf{N}_t + \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega} \ln \mathbf{O}_t.$$

Therefore

$$\ln \mathbf{O}_t = \left(\mathbf{E} - \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^{-1} \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \ln \mathbf{o} + \ln \mathbf{A}_t + \mathbf{Diag}\left(\boldsymbol{\alpha}^k\right) \ln \mathbf{K}_t + \mathbf{Diag}\left(\boldsymbol{\alpha}^l\right) \ln \mathbf{N}_t \right).$$

Meanwhile, combining equations (2.3) and (B.26) yields

$$X_{jt} = \frac{\varphi_j}{\gamma_j} O_{jt}.$$

In turn, we have

$$\ln \mathbf{X}_t = \ln \mathbf{x} + \ln \mathbf{O}_t,$$

where a typical element of $\mathbf{x}$ is $x_j = \varphi_j/\gamma_j$. Consequently,

$$\ln Y_t = \boldsymbol{\varphi}' \ln \mathbf{X}_t = \boldsymbol{\varphi}' \left(\ln \mathbf{x} + \ln \mathbf{O}_t\right) = \ln \overline{A}_t + \boldsymbol{\gamma}' \boldsymbol{\alpha}^k \ln K_t + \boldsymbol{\gamma}' \boldsymbol{\alpha}^l \ln N_t, \tag{B.33}$$

where

$$\boldsymbol{\gamma} \equiv \left(\mathbf{E} - \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}'\right)^{-1} \boldsymbol{\varphi},$$

and

$$\ln \overline{A}_t = \boldsymbol{\varphi}' \ln \mathbf{x} + \boldsymbol{\gamma}' \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \ln \mathbf{o} + \boldsymbol{\gamma}' \ln \mathbf{A}_t + \boldsymbol{\gamma}' \mathbf{Diag}\left(\boldsymbol{\alpha}^k\right) \ln \boldsymbol{\kappa}_{jt} + \boldsymbol{\gamma}' \mathbf{Diag}\left(\boldsymbol{\alpha}^l\right) \ln \boldsymbol{\lambda}_{jt}.$$

Proof.

$$\begin{aligned}
\boldsymbol{\gamma}' \boldsymbol{\alpha}^k + \boldsymbol{\gamma}' \boldsymbol{\alpha}^l &= \boldsymbol{\gamma}' \left(\boldsymbol{\alpha}^k + \boldsymbol{\alpha}^l\right) && \text{(B.34)} \\
&= \boldsymbol{\varphi}' \left(\mathbf{E} - \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^{-1} \left(\mathbf{1} - \boldsymbol{\alpha}^s\right) && \text{(B.35)} \\
&= \sum_{n=0}^{+\infty} \boldsymbol{\varphi}' \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^n \mathbf{1} - \sum_{n=0}^{+\infty} \boldsymbol{\varphi}' \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^n \boldsymbol{\alpha}^s && \text{(B.36)} \\
&= \boldsymbol{\varphi}' \mathbf{1} + \sum_{n=1}^{+\infty} \boldsymbol{\varphi}' \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^{n-1} \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega} \mathbf{1}\right) - \sum_{n=0}^{+\infty} \boldsymbol{\varphi}' \left(\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega}\right)^n \boldsymbol{\alpha}^s. && \text{(B.37)}
\end{aligned}$$

Since $\sum_{j=1}^{S} \omega_{ij} = 1$ for all i, we easily know that

$$\mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \boldsymbol{\Omega} \mathbf{1} = \mathbf{Diag}\left(\boldsymbol{\alpha}^s\right) \mathbf{1} = \boldsymbol{\alpha}^s. \tag{B.38}$$

Moreover, we have normalized that $\sum_{j=1}^{S} \varphi_j = \boldsymbol{\varphi}' \mathbf{1} = 1$. Consequently, we have $\boldsymbol{\gamma}' \boldsymbol{\alpha}^k + \boldsymbol{\gamma}' \boldsymbol{\alpha}^l = 1$. ∎

Denote $\alpha = \boldsymbol{\gamma}' \boldsymbol{\alpha}^k$. Then the above lemma immediately implies that $\boldsymbol{\gamma}' \boldsymbol{\alpha}^l = 1 - \alpha$. In turn, the aggregate output in equation (B.33) can be rewritten as

$$Y_t = \overline{A}_t \left(\prod_{j \in \mathbf{S}} K_{jt}^{\gamma_j \alpha_j^k}\right) N_t^{1-\alpha} = \overline{A}_t \overline{K}_t^{\alpha} N_t^{1-\alpha},$$

where

$$\begin{aligned}
\overline{A}_t &\equiv \exp\left(\boldsymbol{\varphi}' \ln \mathbf{x} + \boldsymbol{\gamma}' \ln \mathbf{o} + \boldsymbol{\gamma}' \ln \mathbf{A}_t + \boldsymbol{\gamma}' \mathbf{Diag}\left(\boldsymbol{\alpha}^l\right) \ln \boldsymbol{\lambda}\right) = \prod_{j \in \mathbf{S}} x_j^{\varphi_j} o_j^{\gamma_j \alpha_j^s} \lambda_j^{\gamma_j \alpha_j^l} \cdot \prod_{j \in \mathbf{S}} A_{jt}^{\gamma_j}, \\
\overline{K}_t &\equiv \left[\prod_{j \in \mathbf{S}} K_{jt}^{\gamma_j \alpha_j^k}\right]^{\frac{1}{\alpha}},
\end{aligned}$$

with

$$
\begin{aligned}
x_j &= \frac{\varphi_j}{\gamma_j}, \\
o_j &\equiv \prod_{i \in \mathbf{S}} \left(\alpha_j^s \omega_{ji} \gamma_j / \gamma_i \right)^{\omega_{ji}},
\end{aligned}
$$

where $\overline{Z} \equiv \prod_{j \in \mathbf{S}} (\varphi_j/\gamma_j)^{\varphi_j} \lambda_j^{\gamma_j \alpha_j^l} o_j^{\gamma_j \alpha_j^s}$. Moreover, we know that

$$
\begin{aligned}
\kappa_{jt} &\equiv \frac{K_{jt}}{\sum_{i \in \mathbf{S}} K_{it}} = \frac{K_{jt}}{K_t}, \\
\lambda_{jt} &\equiv \frac{N_{jt}}{\sum_{i \in \mathbf{S}} N_{it}} = \frac{N_{jt}}{N_t} = \frac{\alpha_j^l \gamma_j}{\sum_{i \in \mathbf{S}} \alpha_i^l \gamma_i} = \lambda_j,
\end{aligned}
$$

where $\boldsymbol{\kappa}_t$ is an $S \times 1$ vector of state variable, and we have used equation (B.27) to simplify λ_{jt}.

Note that in the steady state the relative industry bubble $b_j = \frac{B_j}{Y}$ satisfies:

$$
\chi \bar{b}_j = \sum_{i=1}^{S} \phi_{ij} \xi_j \Gamma_i \bar{b}_i,
$$

where $\chi = \frac{1}{\beta} - 1$, $\bar{b}_j = \sum_{i=1}^{S} \phi_{ij} b_i$, $\Gamma_i = \Gamma(\epsilon_b^*)$. From the bond market clear condition $\sum_j L_j = 0$, we find another equation that b_j satisfies:

$$
\sum_j \theta_j b_j = \frac{\Gamma(\epsilon^*)}{\chi} \sum_{j \in \mathbf{S}} \frac{\frac{\delta x F(x)}{\int_{\varepsilon > x} \varepsilon dF} - \chi - \theta_j^k}{\chi + \delta - \theta_j^k \Gamma(\epsilon^*)} \alpha_j^k \gamma_j.
$$

As a result, the relative industrial bubble b_j depends only on ϵ_b^*. In the meantime, the cutoff ϵ_b^* is determined by:

$$
\epsilon_b^*(\boldsymbol{s}) = \Gamma^{-1} \left(\frac{\chi}{\rho(\mathrm{diag}(\boldsymbol{\theta}_{\boldsymbol{s}}^{\boldsymbol{b}})) \mathcal{M}(s)} \right).
$$

C. Additional Materials

The following figure presents the responses to unexpected bubble bursts without production network.

Figure A.1: Output Loss and Efficiency Cutoff