Organizational Equilibrium with Capital*

Marco Bassetto — Federal Reserve Bank of Chicago

Zhen Huo — Yale University

José-Víctor Ríos-Rull — University of Pennsylvania, UCL, CAERP

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Abstract

This paper proposes a new equilibrium concept – organizational equilibrium – for models with state variables that have a time inconsistency problem. The key elements of this equilibrium concept are: (1) agents are allowed to ignore the history and restart the equilibrium; (2) agents can wait for future agents to start the equilibrium. We apply this equilibrium concept to a quasi-geometric discounting growth model and to a problem of optimal dynamic fiscal policy. We find that the allocation gradually transits from that implied by its Markov perfect equilibrium towards that implied by the solution under commitment, with welfare significantly improved relative to that in the Markov equilibrium. The feature that the time inconsistency problem is resolved slowly over time rationalizes the notion that goodwill is very valuable but has to be built gradually.

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1 Introduction

In this paper we pose an equilibrium concept especially suited for the study of policy settings in macroeconomics where the time inconsistency problem is pervasive and the environment has state variables. Our concept builds upon renegotiation proofness, but adapts it to the challenge of dealing with a dynamic, rather than repeated game. We obtain allocations that are Pareto superior to those of Markov equilibria, but are not supported by trigger-strategy reversion to dominated outcomes.

We argue that equilibria should satisfy three conditions in environments with a sequence of decision makers that see themselves in a similar spot –a form of stationarity even if there are state variables. The first such condition is that any outcome should have the property that no decision maker would rather become an earlier member of the decision making sequence. This *no-restarting condition* limits the use of trigger strategies as a future punishment. A second condition prevents free riding at the start of the process: no agent can do better by sitting out the system (playing Markov) and waiting for future agents to start on a given equilibrium path. This *no-delay condition* is new, to our knowledge, and it prevents jumping to desirable allocations fast. We interpret the implications of this condition as the need for institutions to *slowly earn good will*, like earning a reputation for good behavior without need of unobserved types or triggers. Finally, the third condition imposes optimality within the class of allocations that satisfy the previous two requirements.

Our notion of equilibrium can be seen as a direct extension of that in Prescott and Rios-Rull (2005) and of the notion of *Reconsideration Proofness* in Kocherlakota (1996) to economies with state variables.[1] The extension requires two distinct elements. One is to restrict our attention to a set of environments that display a weak separability property where preferences can be decomposed between a set of actions that we label rescaled actions and the state of the economy. We also provide a strategy to approximate general economies by weakly-separable economies which enables one to explore the

[1]Kocherlakota defines a "state" in his work, but this state only depends on the expectation about current and future actions and is thus purely forward looking. In our case, we define a state as arising from past actions (including possibly past actions of nature, if randomness is present). This is in line with the literature on optimal control and dynamic programming. The role of expectations about current and future actions arises in hybrid environments where some elements of competitive-equilibrium behavior coexist with strategic interactions; we tackle this in Section 5.

organizational equilibrium via solving for the equilibrium of approximated economies (Kubler, 2007). The other element is the inclusion of an additional equilibrium condition that precludes the delay of the implementation of the equilibrium strategy to future agents. This condition has no bite in an environment without state variables, when the payoff of each player is only affected by her actions and those of *future* players. In contrast, we argue that it is a desirable refinement in the case of economies with state variables. It imposes that the coordination that gives rise to the initial equilibrium is not as generous as to tempt the first players to sit out of it, play Markov, and count on the same coordinating mechanism to arise in the future.

Under mild conditions, we prove the existence of an organizational equilibrium. Furthermore, we prove that the organizational equilibrium can be constructed in a recursive way, which greatly simplifies the computation and helps to characterize its properties. The equilibrium converges to a steady state, which is the one preferred by an agent if she could commit future agents to take the same action as she does. From this steady state, the entire transition path then can be solved recursively. Crucially, due to the no-delay condition, agents' actions cannot jump to the steady state immediately, but they converge gradually. On the technical side, in our organizational equilibrium or the reconsideration-proof equilibrium, the approach in Abreu, Pearce, and Stacchetti (1986, 1990) cannot be directly adopted to construct the set of continuation values. We instead develop an alternative method to find this set for the recursive representation, which we think may have its independent value.

We solve for an organizational equilibrium in two benchmark environments. First, we analyze the well studied growth model with quasi-geometric discounting which represents one of the simplest time inconsistency problems (just due to the nature of preferences). Here we show how the economy starts with a very low saving rate and converges to a much higher saving rate, that would have been chosen by any agent if it were to be the constant saving rate for the whole feature. Less simply, but perhaps more interestingly, we solve for the choice of a government that is financed via capital income taxes (other tax instruments can be characterized in essentially similar ways), an environment subject to time inconsistency previously studied by Klein, Krusell, and Ríos-Rull (2008), among others. In both environments the equilibrium allocation is much better (Pareto dominates) than that of the Markov perfect equilibrium. The economy slowly moves towards a high/saving or low taxation behavior, that is, the model slowly overcomes the time consistency problem. We interpret this to be a notion of slowly building reputation, without any need for unobservable types. Here what we call reputation is the result of having displayed in the past a form of patience beyond that implied by the behavior in the Markov perfect equilibrium. We think that this type of behavior helps us understand the value that modern institutions such as governments or central banks pose in showing that they have concerns over the long run, and hence do not take actions such as large capital levies or fast inflationary policies that may have been predicted by models where the present is taken to be the initial period.

Our paper is related to various literatures. It studies macroeconomic environments with time-inconsistency features typically characterized in terms of their Markov equilibria (e.g Cohen and Michel (1988), Currie and Levine (1993), Krusell and Ríos-Rull (1996), Klein and Ríos-Rull (2003), Krusell, Kuruscu, and Smith (2010), Klein, Quadrini, and Ríos-Rull (2005), Bassetto and Sargent (2006), Klein, Krusell, and Ríos-Rull (2008), Bassetto (2008)). It also addresses the type of environments previously studied by posing trigger strategies (Chari and Kehoe (1990), Phelan and Stacchetti (2001)). It is clearly related to the class of models that study versions of the quasi-geometric discounting growth model (Strotz (1956), Phelps and Pollak (1968), Laibson (1997), Krusell and Smith (2003), Chatterjee and Eyigungor (2016), Bernheim, Ray, and Yeltekin (2015), Cao and Werning (2018) among others). Finally, we build on the literature on refinements of subgame perfect equilibrium, particularly in relation to renegotiation proofness (Farrell and Maskin (1989), Kocherlakota (1996), Asheim (1997), Ales and Sleet (2014)).

Two other papers are of special relevance. Like us, Nozawa (2014) also tries to extend the notion of a reconsideration-proof equilibrium to economies with state variables. However, his extension imposes too strict requirements and leads to nonexistence of an equilibrium in many applications. By relying on weak separability, our approach allows us to define "state-free" notions of the economic environment and to establish existence. Brendon and Ellison (2018) analyze optimal policy in the

Ramsey tradition, but they restrict the planner to choose policies that satisfy a recursive Pareto criterion: this criterion disallows sequences that benefit policymakers in the early periods but are dominated for all policymakers from a given time onward. Like them, we also reject policies that allow early decision makers to dictate future paths that lead to early benefits purely at the expense of future decision makers. Rather than developing an optimality criterion, we propose a solution concept aimed at positive analysis, where implicit cooperation across policymakers at different times builds over time. Because of this different motivation, our "no-restarting condition" is imposed on a period-by-period basis. The presence of state variables causes problems in their environment as well, and our approach based on weak separability could be fruitfully applied there too.[2]

We start by posing the issues with time inconsistent preferences in the context of the well understood quasi-geometric discounting growth model with log utility and full depreciation in Section 2. We define organizational equilibrium for separable economies in Section 3, where we also describe the connections to game theory. Section 4 describes a class of examples of such economies and we then define a strategy to study non separable economies via this class of approximation using separable economies. We study the implications of organizational equilibrium for public policy in environments with time-consistency problems in Section 5, adapting our concept to hybrid settings of competitive and strategic behavior. Section 6 concludes.

2 Organizational Equilibrium in a Quasi-Geometric Discounting Growth Model

To provide intuition, we explore the concept of organizational equilibrium in a single-agent decision problem with time-inconsistent preferences, the canonical growth model with quasi-geometric discounting, log utility and full depreciation. This allows us to abstract from considerations relating to the competitive equilibrium emerging from the interaction of many agents, which we analyze in Section 5.

[2] Our approach encompasses the more specific cases introduced by Brendon and Ellison in their latest version to account for state variables.

Assume that the production function is

$$f(k_t) = k_t^{\alpha},$$

and the period utility function is

$$u(c_t) = \log c_t.$$

The law of motion for the state is

$$k_{t+1} = f(k_t) - c_t.$$

The lifetime utility for the agent at period t is given by

$$\Psi_t = u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(c_{t+\tau}\right).$$

It is easy to see that the agent will disagree with herself in the next period if $\delta \neq 1$. For reasons that will be apparent later, we describe the behavior of the household in terms of its saving rates (note that the feasibility of the choice is independent of the level of capital).

2.1 Traditional Notions of Equilibrium in the Quasi-geometric Discounting Economy

Before we discuss the notion of organizational equilibrium, we first characterize the Ramsey outcome (Section 2.1.1) and the differentiable Markov equilibrium (the Markov equilibrium that is the limit of finite economies) (Section 2.1.2).

2.1.1 Ramsey Outcome in the Quasi-geometric Discounting Economy

Suppose that the agent can commit to a particular sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$ at time 0, then the problem of the agent at time 0 is

$$\max_{\{s_t\}_{t=0}^{\infty}} u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t \, u\left(c_t\right),$$

subject to

$$k_{t+1} = s_t k_t^{\alpha},$$

$$c_t = (1 - s_t) k_t^{\alpha},$$

$$k_0 \text{ given.}$$

This problem can be broken into two components: choosing s_0 and choosing $\{s_t\}_{t=1}^{\infty}$. Given s_0, the maximization with respect to future saving rates is a standard recursive problem whose solution has a closed form that is given by the following value function:

$$\Omega(k) = \frac{1}{1-\beta}\left[\log(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\log(\alpha\beta)\right] + \frac{\alpha}{1-\alpha\beta}\log k.$$

This value function is associated with an optimal saving rate which is constant at $s_t = s^R = \alpha\beta$. The Ramsey problem reduces to

$$\max_{s_0} \ u[(1-s_0)k_0^{\alpha}] + \delta\ \beta\ \Omega(s_0 k_0^{\alpha}),$$

and optimal choice of initial saving rate s_0 is

$$s_0 = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}.$$

The initial agent discounts the future more heavily than her future selves, so she is willing to apply a lower saving rate than those in the future, $s_0 < \alpha\beta$. In summary, the sequence of saving rates is

$$s_t = \begin{cases} \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}, & t = 0 \\ \alpha\beta, & t > 0 \end{cases} \tag{2.1}$$

The long run capital stock in the Ramsey problem is $k^R = (\alpha\beta)^{\frac{1}{1-\alpha}}$.[3]

[3] This level of capital is not strictly a steady state since the Ramsey allocation starting from this level of capital will reduce the level of capital before asymptotically returning to it.

2.1.2 Markov Equilibrium in the Quasi-geometric Discounting Economy

We focus on the Markov equilibrium which is continuously differentiable, i.e., it satisfies the generalized Euler equation (GEE). Let $g(k)$ denote the policy function for tomorrow's capital k', the GEE is

$$u_c(f(k) - g(k)) = \beta\, u_c\Big(f[g(k)] - g[g(k)]\Big)\Big[\delta f_k[g(k)] + (1-\delta)\, g_k[g(k)]\Big],$$

which yields

$$g(k) = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}k^{\alpha}.$$

This Markov equilibrium displays a constant saving rate

$$s^M = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}. \tag{2.2}$$

Note that the saving rate in the Markov equilibrium is the same as the first period's saving rate in the Ramsey outcome. The Markov equilibrium has a steady state $k^M = \left(\frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}\right)^{\frac{1}{1-\alpha}} < k^R$.

To characterize agents' payoff, it is useful to introduce the following auxiliary object. If an agent start with capital k and the saving rate for herself and all future selves is a constant s, her lifetime utility is given by

$$\frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log s. \tag{2.3}$$

Define the second part related to the saving rate as

$$\mathcal{H}(s) \equiv \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log s. \tag{2.4}$$

Note that $\mathcal{H}(s)$ is not monotonic in s, and it achieves its maximum at the Ramsey long-run solution s^R only when $\delta = 1$, since the solution s^R disregards the concern for the short run implied by $\delta < 1$. Utilizing equation (2.4), the payoff in the Markov equilibrium for an agent with capital k can be written as

$$\Phi^M(k) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + \mathcal{H}(s^M). \tag{2.5}$$

2.2 Organizational Equilibrium in the Quasi-geometric Discounting Economy

Before we start describing the notion of organizational equilibrium, we establish an important property of this economy, that preferences of agents over the inherited capital stock and any sequence of savings rates display separability, *i.e.* that the utility can be written as a function of the initial capital and of an aggregate that depends only on the sequence of savings rates (Section 2.2.1). Then we define an organizational equilibrium in terms of savings rates, and we construct it, therefore establishing existence in Section 2.2.2.

2.2.1 Separability

Consider a sequence of savings rates $\{s_t\}_{t=0}^{\infty}$ and an initial capital stock k_0. The implied sequence of capital stocks, $\{k_t\}_{t=0}^{\infty}$, has the property that $k_t = k_0^{\alpha^t}\Pi_{j=0}^{t-1}s_j^{\alpha^{t-j-1}}$. Accordingly, the lifetime utility for the agent with capital k_0 is

$$\begin{aligned}
&U(k_0, s_0, s_1, \ldots) \\
&= \log(1-s_0)k^{\alpha} + \delta\sum_{j=1}^{\infty}\beta^j \log[(1-s_j)k_j^{\alpha}] \\
&= \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_0 + \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_0) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_j) + \frac{\alpha\beta}{1-\alpha\beta}\log(s_j)\right).
\end{aligned}$$

The same logic follows for the date-t agent which means that her lifetime utility, or total payoff, is the sum of a term that depends on the period t capital and a term that depends only on saving rates of periods t and after. We write it compactly as

$$\underbrace{U(k_t, s_t, s_{t+1}, \ldots)}_{\text{total payoff}} = \underbrace{\frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t}_{\text{capital payoff}} + \underbrace{V(s_t, s_{t+1}, \ldots)}_{\text{action payoff}} \tag{2.6}$$

We also write more compactly the total payoff into the term that depends on the state and the term that depends on the subsequent actions

$$v(k,V) := \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + V$$

where

$$V(s_0,s_1,\ldots) := \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_0) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_j)+\frac{\alpha\beta}{1-\alpha\beta}\log(s_j)\right).$$

In this economy, the preferences as of time t are separable in the level of capital k_t and the sequence of current and future saving rates. In fact, that the terms in k_0 and V are additive implies a *strong* form of separability. For our purposes, only a weak form of separability is required as we will see in Section 3.

2.2.2 Discussion of Organizational Equilibrium

We now look at the features that the organizational equilibrium should have. We exploit separability to specify how to run comparisons across agents with differing initial conditions: specifically, we impose that agents factor out the component of utility arising from initial capital, and evaluate proposals based on the sequence of saving rates alone, looking at the subutility V. On this subutility, we impose the requirements that no agent would prefer being a previous member of the sequence and that no agent would have an incentive to wait for a proposal to be implemented starting from the next period. Formally,

Definition 1. *A sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$ is organizationally admissible if*

1. *$V(s_t,s_{t+1},s_{t+2},\ldots)$ is (weakly) increasing in t.*

2. *The first agent has no incentive to delay the proposal.*

$$V(s_0,s_1,s_2,\ldots) \geq \max_s V(s,s_0,s_1,s_2,\ldots)$$

Within organizationally admissible sequences, any sequence that attains the maximum of $V(s_0, s_1, s_2, \ldots)$ *is an* organizational equilibrium*.*

Here, we use a heuristic approach to illustrate the key insight, and a more formal proof can be found in Appendix and Section 3. We proceed in three steps to show the role played by each of the conditions in Definition 1.

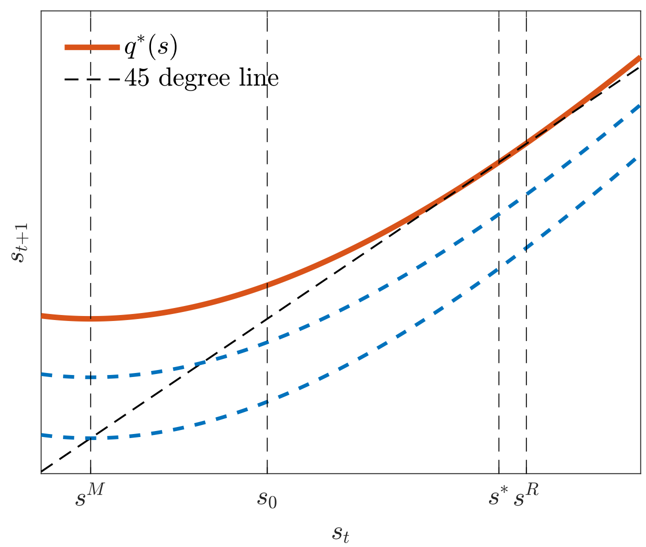
No-restarting Condition The first requirement is that no agent would prefer to become a previous member of the sequence. If we ignore the second no-delay condition for a moment, then the action payoff has to be some constant, i.e., $V(s_t, s_{t+1}, \ldots) = \overline{V}$. Otherwise, there exists some $j > 0$ such that $V(s_0, s_1, \ldots) < V(s_j, s_{j+1}, \ldots)$, and the initial agent would like to implement a different sequence starting from s_j to enjoy a higher utility. This constant action payoff condition leads to a first-order difference equation between two consecutive saving rates

$$(1-\beta)\overline{V} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - \beta(1-\delta)\log(1-s_{t+1}). \tag{2.7}$$

In Figure 1, blue dash curves correspond to the difference equations with different $\overline{V}$. Note that for each $\overline{V}$, there are a continuum of sequences $\{s_t\}$ that satisfies equation (2.7), indexed by their starting points of the difference equation. Particularly, recall that the action payoff in the Markov equilibrium is $\overline{V} = \mathcal{H}(s^M)$. Therefore, a continuum of sequences yield the same action payoff as the Markov equilibrium. The Markov equilibrium features a constant saving rate (the intercept of the lower blue curve with 45 degree line), while other sequences involve transition dynamics.

Maximization Condition Among different action payoff $\overline{V}$, the initial agent should choose the best outcome from them. In this quasi-geometric discounting economy, increasing $\overline{V}$ is corresponding to shifting the curve upwards in Figure 1. The highest curve can be chosen is the one that is tangent with the 45 degree line (the red curve), and we denote its associated action payoff as V^*. Any $\overline{V} > V^*$ will result in the sequence of saving rates diverging to one, which cannot be optimal. This

FIGURE 1: Proposal Function $q^*(s)$



tangent point s^* is a fixed point of the difference equation, which turns out to be the level of saving rate that maximizes the function $\mathcal{H}(s)$. That is, s^* achieves the highest steady state action payoff, and it is easy to verify that $s^M < s^* < s^R$ when $\delta < 1$.

No-Delay Condition So far, we have not taken the no-delay condition (the second condition in Definition 1) into account. The choice of V^* permits a continuum of sequences of saving rates, and only a subset of them satisfy the no-delay condition. From the perspective of the agent who makes the proposal, all of these sequences of saving rates yield the same payoff for herself; however, they yield different payoffs from the perspective of future agents. As an example, consider the following two proposals: the first one is $s_t = s^*$ for all t, and the second one starts from $s_0 = s^M$ and subsequently follows a sequence dictated by the difference equation (2.7). Both of these sequences imply the same action payoff $V(s_0, s_1, \ldots) = V(s^*, s^*, \ldots) = V^*$ for the agent who makes the proposal, as a higher saving rate s^* today is rewarded with higher saving rates s^* in the future. One potential solution would be for agents to coordinate on the Pareto preferred s^*. However, under this criterion, it would logically follow that future agents would always have an incentive to disregard the past and coordinate on Pareto-preferred outcomes; in this case, the current agent would have an incentive to delay, play $s = s^M$, and count on next period's agent to start the constant s^* sequence. We require instead that s_0 is such that $V(s^M, s_0, s_1, \ldots) \leq V^*$, so that the initial agent would not gain from waiting for the sequence to start in the next period. To guarantee the no-delay condition is satisfied, the sequence of saving rates has to start with a low enough point to prevent the initial agent from sitting out. This condition excludes the possibility of jumping to the steady-state saving rate immediately, and a gradual transition has to take place.

The following proposition summarizes previous arguments and provides the analytical solution.

Proposition 1. *There exist organizational equilibria. In any such equilibrium, the evolution of the saving rate is given recursively by the proposal function* q^*

$$s_t = q^*(s_{t-1}) = 1 - \exp\left\{\frac{-(1-\beta)V^* + \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_{t-1} + \log(1-s_{t-1})}{\beta(1-\delta)}\right\} \tag{2.8}$$

where the fixed point saving rate $s^* = q^*(s^*)$ *and the maximum utility* V^* *are given by*

$$s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta} \quad \text{and} \tag{2.9}$$

$$V^* = \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s^*) + \frac{\alpha\delta\beta}{(1-\beta)(1-\alpha\beta)}\log s^*. \tag{2.10}$$

Equilibria differ by their initial saving rate, which belongs to the interval

$$s_0 \in \left[\underline{s}, q^*\left(s^M\right)\right], \tag{2.11}$$

where $\underline{s} = \min\left\{s : \frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) = (1-\beta)V^* + \beta(1-\delta)\log(1-s^*)\right\}$.

While there are many organizational equilibria, all of whom give the same utility to the first generation, the equilibrium in which

$$s_0 = q^*\left(s^M\right)$$

yields the highest total utility for the subsequent generation by delivering the most capital. We select this one because it is the natural outcome if there is an arbitrarily small amount of altruism involved.

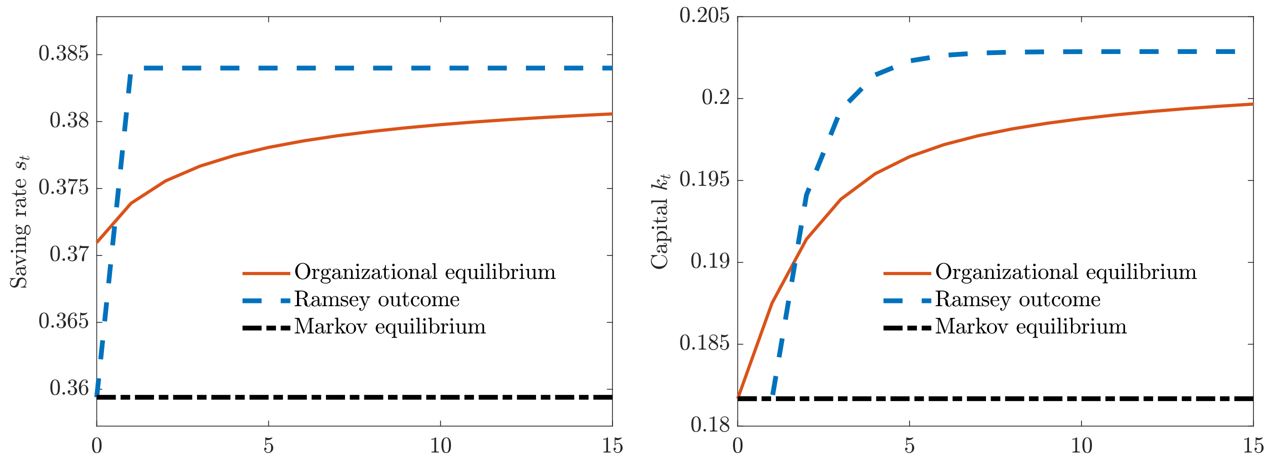
In the organizational equilibrium, time inconsistency is gradually overcome through time: at least from period 2 on, the saving rate exceeds that of the Markov equilibrium, and a virtuous cycle is started, with a monotonic increase which converges to s^*. Initial saving is limited by the temptation to let the next generation start the virtuous cycle. This temptation diminishes in subsequent periods, since restarting the virtuous cycle from scratch implies giving up on the accumulated effect of previous increases in s_t. Note that s^* is below the long-term savings rate of the Ramsey outcome, no matter how close to 1 δ is (as long as it is strictly less than 1): while the equilibrium path converges to the Ramsey outcome as $\delta \to 1$, it never coincides with it, and the folk theorem does not apply. The saving rate s^* is the preferred constant saving rate of the agents, weighing their current patience with their future impatience. This is illustrated in Figure 1.

2.3 Comparison with Other Equilibria

We now compare the properties of the organizational equilibrium of the sequence of capital and the lifetime utilities with those in the Ramsey outcome and the Markov equilibrium.

We first turn the transition paths of different equilibria. We assume that the initial capital stock is the steady state capital stock in the Markov equilibrium, i.e., $k_0 = k^M$. Figure 2 displays the transition paths for the saving rate s_t and capital k_t. In the Markov equilibrium, the capital stock remains unchanged at its steady-state level which we assumed as a starting point. The Ramsey outcome features the same saving rate as the Markov equilibrium in the first period, so that the capital stock remains the same at the beginning of the second period. From the second period onwards the saving rate increases to s^R permanently. The sequence of saving rates in the organizational equilibrium is induced by the proposal function $s_{t+1} = q^*(s_t)$. Particularly, the saving rate in the first period is $s_0 = q^*\left(s^M\right) > s^M$, and the capital is initially higher than in the Ramsey allocation. Over time, the saving rates increases gradually and converge to $s^* < s^R$. Asymptotically, capital in the

Figure 2: Transition Path I: Allocation



organizational equilibrium settles between the Ramsey outcome and the Markov equilibrium.

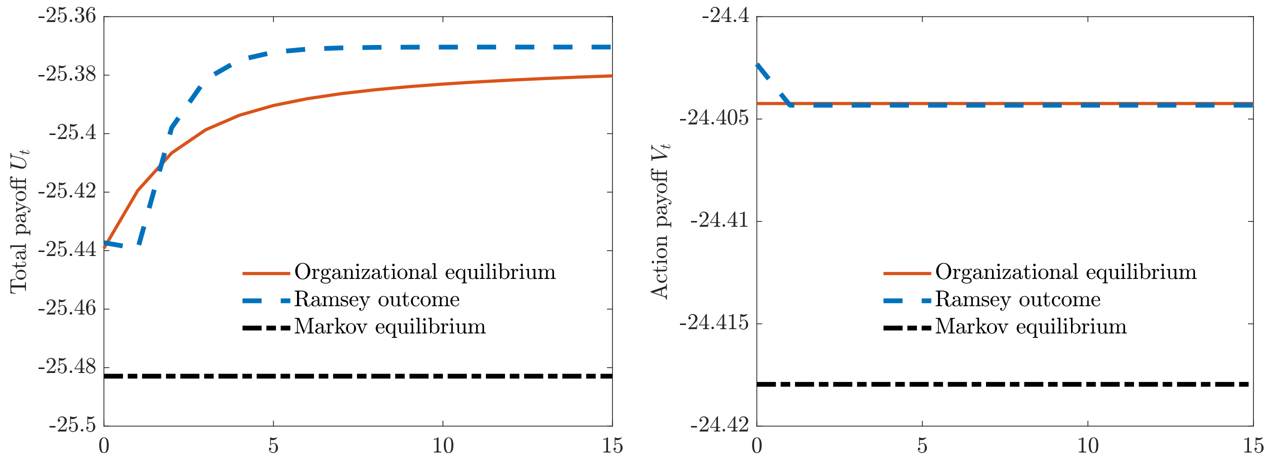
Now we turn to the welfare comparison. Given a particular sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$, based on the analysis in the last section, the lifetime utility for generation t can be written as

$$\underbrace{U_t(k_t, \{s_\tau\}_{\tau=t}^{\infty})}_{\text{total payoff}} = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + \underbrace{V_t}_{\text{action payoff}}.$$

The total payoff U_t and the action payoff V_t are depicted in Figure 3. The total payoff in the Markov equilibrium is the lowest during the entire transition, which is the result of both the lowest capital stock and action payoff.

The comparison between the Ramsey outcome and the organizational equilibrium is more subtle. In the first period, the total payoff in the Ramsey outcome is higher than that in the organizational equilibrium: this has to happen by definition, since the Ramsey outcome maximizes the total payoff from the perspective of period 0. In the following period, the comparison reverses, and the total payoff in the organizational equilibrium is actually higher than the Ramsey outcome. This happens both because the initial generation accumulates additional capital, and because the organizational

Figure 3: Transition Path II: Payoff



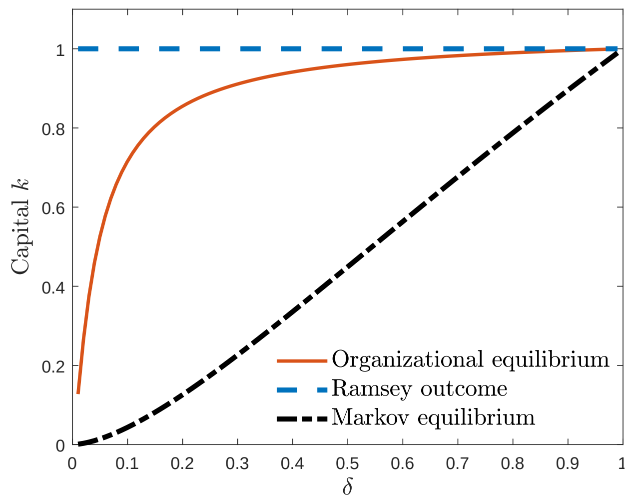
equilibrium does not impose as high a saving rate, allowing for some indulgence for the short-run impatience that arises in the second period. Our notion of organizational equilibrium treats initial capital as a bygone, factoring it out of the payoff that is relevant in computing the equilibrium itself; however, it captures the notion that the initial agent is not privileged compared to future decision makers and cannot impose on them sacrifices that she has not undertaken. For this reason, when we focus on V_t, an organizational equilibrium redistributes from the initial agent to all future decision makers. When comparing the total payoff, after period 0, early decision makers benefit both from a higher capital level and a higher action payoff, while eventually capital falls below the Ramsey outcome and late generations lose from this.

In terms of the steady state capital level, Figure 4 shows how it changes with δ. The capital stock in the organizational equilibrium (k^O), the Markov equilibrium (k^M), and the Ramsey outcome (k^R) are related by the following simple ratios:

$$\frac{k^O}{k^R} = \left(\frac{1}{\frac{1}{\delta} - \beta(1-\alpha\beta+\alpha)\left(\frac{1}{\delta}-1\right)} \right)^{\frac{1}{1-\alpha}} \text{ and } \frac{k^M}{k^O} = \left(1 - \frac{\beta(1-\delta)(1-\alpha\beta)}{1-\alpha\beta+\delta\alpha\beta}\right)^{\frac{1}{1-\alpha}}.$$

In the Ramsey outcome, the stationary allocation is independent of δ, and we normalize it to 1. As

FIGURE 4: Comparison of Stationary Allocations



can be seen from the Figure, as δ decreases, the capital stock in the Markov equilibrium decreases faster than that in the organizational equilibrium.

3 Organizational Equilibrium: A General Definition

We proceed now to define organizational equilibrium in a more general manner. The spirit of organizational equilibrium is that the solution concept should not treat the current decision maker more favorably than future ones. This notion requires some form of stationarity.

Consider a generic environment of sequential decision makers (typically those that have a time-consistency problem) where there is a physical state variable $k \in K$. Specifically, given the current level of k, the agent making a decision will choose an action a from a set A. The state evolves according to $k_{t+1} = F(k_t, a_t)$. Preferences for an agent making decisions in period t are given by $U(k_t, a_t, a_{t+1}, a_{t+2}, \ldots)$. The first assumption is that functions U and F are independent of calendar time, which allows meaningful welfare comparisons across decision makers.

In the absence of a state variable, the economy looks the same starting at any time t, and "not treating more favorably" the current decision maker readily translates to not offering to her higher utility than to other decision makers. However, when a state variable is present, imposing the same utility becomes an unnatural restriction: as an example, as capital evolves, the sequences of consumption which can be supported by the given capital change. We follow here an alternative approach by restricting the environments that we study to those in which the utility is weakly separable between the state and the sequence of actions, such that the preference ordering over sequences actions is independent of the initial state. It is then natural to require that the equilibrium choice of actions be independent of the state. This amounts to a form of weak separability in terms of utility between the state and the actions. Formally the assumptions on the environment that we make are:

Assumption 1. 1. *At any point in time t, the set of feasible actions A is independent of the state k_t;*

2. *U is weakly separable in k and in $\{a_s\}_{s=0}^{\infty}$, i.e., there exist functions $v : K \times \mathbb{R} \to \mathbb{R}$ and $V : A^{\infty} \to \mathbb{R}$ such that*

$$U(k, a_0, a_1, a_2, \ldots) \equiv v(k, V(a_0, a_1, a_2, \ldots)). \tag{3.1}$$

and such that v is strictly increasing in its second argument.

Sometimes the original problem does not satisfy Assumption 1, but it is possible to rescale actions in such a way that it does. As an example, the original specification of the saving problem with quasi-geometric discounting does not satisfy Assumption 1 if we define the action to be consumption: the feasible set of consumption levels depends on initial capital.[4] Formally, suppose that the set of feasible actions at any capital level k is $\tilde{A}(k) \subseteq \tilde{A}$ and that preferences are given by $\tilde{U}(k_t, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots)$. Our construction still applies as long as it is possible to find a set of actions A and a function γ such that $\tilde{a} = \gamma(a, k)$ and that Assumption 1 holds for A, where

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) \equiv \tilde{U}(k, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots),$$

[4]Note that weak separability automatically fails if certain actions are only feasible for some levels of capital, since, holding actions fixed, the left-hand side of (3.1) would then be well defined for some values of k and not for others.

and where for $t \geq 0$, $\tilde{a}_t$ is computed recursively as

$$\begin{aligned} \tilde{a}_t &= \gamma(a_t, k_t), \\ k_{t+1} &= F(k_t, \tilde{a}_t). \end{aligned} \tag{3.2}$$

We are now ready to define organizational equilibrium.

Definition 2. *A sequence of actions $\{a_t\}_{t=0}^{\infty}$ is organizationally admissible if it satisfies the following requirements:*

1. *$V(a_t, a_{t+1}, a_{t+2}, \ldots)$ is (weakly) increasing in t; this condition ensures that subsequent agents would not choose to rewind time.*
2. *The first agent has no incentive to delay the proposal.*

$$V(a_0, a_1, a_2, \ldots) \geq \max_{a \in A} V(a, a_0, a_1, a_2, \ldots); \tag{3.3}$$

Within organizationally admissible sequences, any sequence that attains the maximum of $V(a_0, a_1, a_2, \ldots)$ is an organizational equilibrium*.*

3.1 Game-Theoretic Foundations and Relation to Other Equilibrium Notions

In this section, we discuss the connection between an organizational equilibrium and related notions of equilibria in games. Our notion is most closely related to Kocherlakota's (1996) reconsideration-proof equilibrium. While the two notions are very similar, two main differences emerge, which we will discuss in turn:

- By exploiting weak separability, an organizational equilibrium extends the notion of reconsideration proofness to dynamic games, rather than purely repeated games;
- In an environment without state variables, all reconsideration-proof equilibria have the same value for all players. This is no longer the case for organizational equilibria. Our no-delaying

condition takes into account the role of the state variable to select a subset of the reconsideration-proof equilibria. This selection is in line with the original motivation of renegotiation (and reconsideration) proofness, but it imposes it in a limited way which still allows for existence of an equilibrium.

The general setup that we introduced in this section represents a game, with an infinity of players indexed by the time at which they act (0,1,...), each of whom has preferences given by (3.1). At each time t, the history of play is given by $h^t := (a_0, a_1, ...a_{t-1})$, with $h^0 := \emptyset$. A strategy σ_t for player t is a mapping from the set of time-t histories, H^t, to the set of actions A. A strategy profile is a sequence of strategies, one for each player: $\sigma := (\sigma_0, \sigma_1, ...)$. As usual, it is also convenient to define a continuation strategy after history h^t, $\sigma|_{h^t}$, represented by the restriction of $(\sigma_t, \sigma_{t+1}, ...)$ to the histories following h^t. So far, we have defined an organizational equilibrium only by its equilibrium path. The simplest way to fully specify the game strategy that supports it as a subgame-perfect equilibrium is to use the no-delaying condition (3.3) and set $\sigma|_{h^t} = \sigma$ whenever

$$a_{t-1} \neq \sigma_{t-1}(h^{t-1}), \tag{3.4}$$

where $h^t = (h^{t-1}, a_{t-1})$. According to this strategy, the equilibrium of the game prescribes restarting from the equilibrium path of period 0 whenever a deviation occurs. This is not the only possibility; as an example, the function q^* defined in (2.8) can be used in the quasi-geometric saving problem to recursively generate an alternative strategy (not just an equilibrium path) that supports the organizational equilibrium as a subgame-perfect equilibrium.

When there are no state variables the game presented here is encompassed by those considered in Kocherlakota (1996).[5] Kocherlakota analyzes a purely forward-looking environment, in which the

[5]Kocherlakota defines a "state" in his work, but this state only depends on the expectation about current and future actions, which makes it really not a state. In our case, we define a state as arising from past actions (including possibly past actions of nature, if randomness is present). This is in line with the literature on optimal control and dynamic programming. Our analysis can be extended straightforwardly to situations in which expectations about current and future actions affect the current set of actions and payoffs, as it happens in hybrid environments where some elements of competitive-equilibrium behavior coexist with strategic interactions.

payoff accruing to the player in period-t only depends on the actions (or the expectations about the actions) of players in period t onwards. In this environment, he defines a subgame-perfect equilibrium to be symmetric if its continuation value is independent of the past history of play. A reconsideration-proof equilibrium is then an equilibrium that achieves the highest payoffs within symmetric equilibria.

To prove existence of an organizational equilibrium and to link with reconrideration-proof equilibrium, we proceed in two steps. First, we rely on Kocherlakota (1996) to prove the existence of a reconsideration-proof equilibrium for the game with payoff function $V(.)$. We then show that there exits a path such that the threat of restarting from the period-0 path after a deviation is a sufficient deterrent, no matter which deviation a player is considering.

The following assumption mirrors Kocherlakota's:

Assumption 2. *1. A is a convex compact subset of a locally convex topological linear space with topology ρ_x.*

2. V is quasiconcave over A^∞.

3. V is continuous over A^∞ with respect to the product topology ρ_x^∞.

Under Assumption 2, Proposition 4 in Kocherlakota (1996) proves that a reconsideration-proof equilibrium exists for the game whose period-t payoff is $V(a_t, a_{t+1}, a_{t+2}, ...)$. This equilibrium achieves the maximal utility within symmetric equilibria.

To make sure the no-delay condition can be satisfied, we further assume that the value function V is separable between its first and subsequent arguments.

Assumption 3. *V is in turn weakly separable in a_0 and $\{a_s\}_{s=1}^\infty$, i.e., there exist functions $\widetilde{V}$: $A \times \mathbb{R} \to \mathbb{R}$ and $\widehat{V} : A^\infty \to \mathbb{R}$ such that, for all sequences $(a_0, a_1, a_2, \dots) \in A^\infty$,*

$$V(a_0, a_1, a_2, ...) = \widetilde{V}(a_0, \widehat{V}(a_1, a_2, \dots)),$$

with $\widetilde{V}$ strictly increasing in its second argument.

The following proposition establish the existence of organizational equilibrium and shows that an organizational equilibrium is reconsideration proof in the absence of state variables.

Proposition 2. *Under Assumptions 2 and 3, consider a game where time-t preferences are given by*

$$V(a_t, a_{t+1}, a_{t+2}, \ldots).$$

1. *There exists an organizational equilibrium.*
2. *If a path $(a_0, a_1, \ldots)$ is an organizational equilibrium, then it is the outcome of a reconsideration-proof equilibrium.*

Proof. We first show that there exits a reconsideration-proof equilibrium that satisfies condition 2 in Definition (2). Let $(a_0^E, a_1^E, \ldots)$ be the outcome of a reconsideration-proof equilibrium for the game whose period-t payoff is $V(a_t, a_{t+1}, a_{t+2}, ...)$, and let V^* be its associated value. This means that, for any period t and any actions $a \in A$, there exists a continuation sequence $(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)$ which is also a reconsideration-proof equilibrium and is such that

$$V(a_t^E, a_{t+1}^E, a_{t+2}^E, \ldots) \geq V(a, a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots). \tag{3.5}$$

We then have

$$V(a, a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots) = \widetilde{V}(a, \widehat{V}(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)).$$

Acknowledging that the sequence $(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)$ is potentially a function of the deviation a (as well as of time t, which we can hold fixed), define

$$\underline{V} := \inf_{a \in A} \widehat{V}(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots). \tag{3.6}$$

By the compactness of A, Tychonoff's theorem, and continuity of $\widehat{V}$, we can find a sequence of actions

$(a_0^*, a_1^*, ...)$ that attains the infimum in equation (3.6) above. Exploiting Assumption 3, this sequence ensures subgame perfection and satisfies the no-delay condition:

$$V(a_0^*, a_1^*, a_2^*, \ldots) \geq V(a, a_0^*, a_1^*, \ldots).$$

This path attains a value V^*.

Now we show that $(a_0^*, a_1^*, ...)$ is also an organizational equilibrium. Consider the set of sequences $\{a_\tau\}_{\tau=0}^{\infty}$ that satisfy the condition that lifetime utility is weakly increasing (condition 1 in Definition 2). Any sequence $\{a_\tau\}_{\tau=0}^{\infty}$ that attains the maximum of $V(a_0, a_1, \ldots)$ within this set has to be such that $V(a_t, a_{t+1}, \ldots) = \overline{V}$ for any t. If not, then there exists t such that $V(a_t, a_{t+1}, \ldots) < V(a_{t+1}, a_{t+2}, \ldots)$. The initial proposer can copy the sequence starting from agent in period $t+1$ by proposing $\{\widehat{a}_\tau\}$ where $\widehat{a}_\tau = a_{\tau+t+1}$. If $\overline{V}$ cannot be attained in a symmetric equilibrium, then $\{a_\tau\}_{\tau=0}^{\infty}$ cannot satisfy the no-delay condition. Otherwise, the strategy of restarting from the beginning after any deviation will support $\{a_\tau\}_{\tau=0}^{\infty}$ as the outcome of a symmetric equilibrium. Therefore, the value attained in an organizational equilibrium cannot be larger than the largest value attained by a symmetric equilibrium, V^*. We have shown that $(a_0^*, a_1^*, ...)$ attains the value V^* and it satisfies condition 2 in Definition 2 as well, which is therefore an organizational equilibrium for the game without state variables. By weak separability (Assumption 1), this property carries over to the original game with state variables. Hence, playing $(a_0^*, a_1^*, ...)$ is an organizational equilibrium.

Finally, if $(a_0, a_1, \ldots)$ is an organizational equilibrium, it must be that $V(a_t, a_{t+1}, \ldots) = V^*$ for any $t > 0$, otherwise $(a_0^*, a_1^*, ...)$ cannot be an organizational equilibrium. Consider the strategy that any deviation from the prescribed play leads to a restart of the path. This strategy supports $(a_0, a_1, \ldots)$ as a reconsideration-proof equilibrium, which completes the proof. □

In the presence of a state variable, an organizational equilibrium assumes that players coordinate on strategies which only depend on the history of play h^t and not on the physical state. In this case, an organizational equilibrium imposes symmetry only in that the payoff of the subutility V

is independent of the history of play, but the payoff of each time-t player is still different across histories which lead to different levels of the state. Intuitively, a different state implies a different set of possible utility levels going forward, so we should expect it to affect payoffs in the subgames going forward. However, this dependence of utility from the state takes a simple form under weak separability, and there is a natural mapping across histories with different levels of capital: the same sequences of actions are possible under any level of capital, and the preferences of player t over the sequences from date t on are also represented by the subutility V, independent of k_t. For this reason, imposing reconsideration proofness on preferences represented by V alone is appealing.

Our construct cannot of course completely get around the presence of a physical state, and it is for this reason that the no-delaying condition has some bite. In a reconsideration-proof equilibrium, where the state is not there, at any time t both the current and the *future* players' equilibrium payoffs are independent of the past history of play. This is no longer true in an organizational equilibrium when the state matters: while the current player receives the same equilibrium payoff for all histories that share the same state, future players do not.

To illustrate this point concretely, consider the example of Section 2. At each point t, the equilibrium payoff for player t depends on the past history of play only through k_t and not through the entire past history of actions; moreover, the equilibrium is such that player t is indifferent between all saving rates in $\left[q^*\left(\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}\right), s^*\right]$. However, *future* players would strictly prefer the equilibrium path that would unfold if player t chose s^*, which would lead to s^* being played for ever. If we appealed to (strong) Pareto optimality to select among equilibria, then s^* would be selected. But this equilibrium is suspect for the same logic that leads us to discard the trigger strategies that support the best subgame-perfect equilibrium for player t. Specifically, if player t anticipates that, as of $t+1$, players will coordinate on the Pareto-optimal equilbrium and will thus play s^* independently of past history, she has an incentive to play the best one-shot saving rate instead. Our no-delaying condition imposes that, whatever coordination mechanism selects the equilibrium to be played from period 0, no player at any time could be better off by deviating and counting on other players to use the same coordination mechanism to restart the game. Formally, given an equilibrium strategy profile σ, we require

$$V(\sigma_t(h^t), a_{t+1,\sigma|_{h^t}}, a_{t+2,\sigma|_{h^t}}, ...) \geq V(\tilde{a}_t, \sigma_0, a_{1,\sigma}, a_{2,\sigma}, ...) \quad \forall \tilde{a}_t \in A, h^t,$$

where we used the following short-hand notation:

$$\begin{aligned} a_{t+1,\sigma|_{h^t}} &:= \sigma_{t+1}(h^t, \sigma(h^t)), \qquad & a_{t+s,\sigma|_{h^t}} &:= \sigma_{t+s}(h^t, a_{t+1,\sigma|_{h^t}}, ..., a_{t+s-1,\sigma|_{h^t}}), \\ a_{1,\sigma} &:= \sigma_1(\sigma_0), & a_{s,\sigma} &:= \sigma_s(\sigma_0, a_{1,\sigma}, ...a_{s-1,\sigma}). \end{aligned}$$

It's useful to compare organizational equilibrium to two alternative notions of equilibrium for dynamic games which are inspired by similar concerns about what constitutes a "credible punishment" in subgame-perfect equilibria.

An extension of reconsideration-proofness to environments with state variables was already proposed by Nozawa (2014). Nozawa requires weakly reconsideration proof equilibria to be such that the equilibria of all subgames share the same payoff *function* $\Psi(k)$, which depends on the state; in the absence of the state, this reduces to Kocherlakota's (1996) symmetry requirement. A strong reconsideration-proof equilibrium is then an equilibrium in which $\Psi(k)$ is undominated by any other equilibrium *point by point*. This is often too strong a requirement, and hence existence may fail. As an example, no reconsideration-proof equilibrium would exist in the example of Section 2.

An alternative approach is revision proofness, which was introduced by Asheim (1997) and made explicit as a game in Ales and Sleet (2014). In their papers, a larger class of credible punishments is allowable. Specifically, under reconsideration proofness, if Σ is the set of equilibrium strategies of the game, each player at any time t is allowed to coordinate current and future play to her favorite element of Σ. Under revision proofness, player t's coordination power is limited because she is required to propose deviations from the equilibrium path of play that benefit *all* future players. The resulting equilibrium set is much larger. For the case of quasi-geometric discounting with linear preferences, Ales and Sleet (2014) show that all subgame-perfect paths better than the Markov equilibrium are revision proof. In environments with state variables, a limitation of revision proofness is that it is unclear how a future player could "block" a revision proposal when she would inherit a different state under the revision proposal and would thus not be able to continue with the original strategy.

Our notion of organizational equilibrium retains the unilateral aspect of deviations from reconsideration proofness, but it relies on weak separability to define and impose symmetry across different levels of capital. The role of Pareto optimality enters in a limited way through the no-restarting condition and potentially through a final selection of a Pareto optimal path among those that satisfy symmetry and no-restarting.

3.2 Recursive Structure

We conclude this section by studying the case in which the function $\widehat{V}$ admits a recursive structure, as is the case in the example of Section 2 and in many other applications of economic interest. This structure in turn provides a useful way to compute and characterize organizational equilibria.

Assumption 4. *There exists a function $W : A \times \mathbb{R} \to \mathbb{R}$, increasing in the second argument, such that, given any sequence $\{a_t\}_{t=0}^{\infty} \in A^{\infty}$,*

$$\widehat{V}(a_0, a_1, a_2, \ldots) \equiv W\left(a_0, \widehat{V}(a_1, a_2, \ldots)\right), \tag{3.7}$$

Proposition 3. *Under Assumptions 3, 2 and 4, there exists an organizational equilibrium $\{a_t\}_{t=0}^{\infty}$ which is recursive in the value $\widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$: that is, there exists a function $g : \mathbb{R} \to A \times \mathbb{R}$ such that $(a_t, v_{t+1}) = g(v_t)$, and $v_t = \widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$ for all $t = 0, 1, \ldots$.*

Proof. See Appendix B. □

Proposition 3 uses values as a state variable in ways similar to Abreu, Pearce, and Stacchetti (1986, 1990). However, as the proof shows, constructing the set of possible values is considerably more involved than in the case of Abreu, Pearce, and Stacchetti; hence, while the proposition greatly simplifies the task of constructing organizational equilibria, it still leaves a challenging task. The following proposition is useful both as a characterization of the properties of an organizational equilibrium, and as an avenue for finding equilibria efficiently.

Proposition 4. *Under Assumptions 3, 2, and 4, the value of* $\widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$ *is increasing over time for any organizational equilibrium, and it converges to the value associated with the steady state which maximizes* $V(a, a, a, ...)$*. Furthermore, if* $\widehat{V}$ *is a strictly quasiconcave function and the steady state that maximizes* $V(a, a, a, ...)$ *is not a Markov equilibrium. Then the initial value of* $\widehat{V}(a_0, a_1, a_2, \ldots)$ *is strictly below the steady state: convergence is not immediate.*

Proof. See Appendix B □

Proposition 4 shows that the procedure we used to compute organizational equilibria in Section 2.2 applies more generally. Specifically, we first compute a steady state that maximizes $V(a, a, a, ...)$: this is the steady state that would be chosen by the decision maker at time 0 if she could commit future players to take the same action. This maximization yield a value V^* which must remain constant along the path, i.e., $V(a_t, a_{t+1}, a_{t+2}, ...) = V^*$. We then construct a path that leads to this steady state, using the no-delay condition to inform us of the starting point. The last part of the proof shows that convergence to the steady state takes time, unless we are in a special case in which the steady state can be supported in a Markov equilibrium with no intertemporal incentives.

4 Approximated Equilibrium

In this section, we first introduce a class of weakly separable economies with the quasi-geometric discounting preferences. This class include the example discussed in Section 2 as a special case, and it also include some other interesting models explored in the literature. We then proceed to provide an approximation algorithm for models that are not weakly separable.

4.1 A Class of Weakly Separable Economies

Consider the following class of economies. The state variable is k and the action is a. The preference is specified as

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} \, u\left(k_{t+\tau}, a_{t+\tau}\right).$$

Suppose that the period utility function takes the form of

$$u(k_t, a_t) = \omega_{10} + \omega_{11} h(k) + \omega_{12} m(a),$$

and the state evolves according to

$$h(k_{t+1}) = \omega_{20} + \omega_{21} h(k_t) + \omega_{22} g(a_t),$$

for some monotonic functions h, g, and m, and some constant matrix $\boldsymbol{\omega}$. In Section 2, the economy is corresponding to $h(k) = \log(k)$, $g(a) = \log(a)$ and $m(a) = \log(1-a)$. It is easy to verify that this class of economies are weakly separable. Given a sequence of actions $\{a_\tau\}_{\tau=0}^{\infty}$ and the initial state k_0, the sequence of state variables follows

$$h(k_t) = \omega_{20} \frac{1-\omega_{21}^t}{1-\omega_{21}} + \omega_{21}^t h(k_0) + \omega_{22} \sum_{\tau=0}^{t-1} \omega_{21}^{t-1-\tau} g(a_\tau),$$

and the lifetime utility is given by

$$\begin{aligned} U(k_0, \{a_\tau\}_{\tau=0}^{\infty}) = & \frac{1-\beta+\delta\beta}{1-\beta} \omega_{10} + \frac{\omega_{11}(1-\beta\omega_{21}+\delta\beta\omega_{21})}{1-\beta\omega_{21}} h(k_0) \\ & + \omega_{12} m(a_0) + \frac{\omega_{11}\omega_{22}\delta\beta}{1-\beta\omega_{21}} g(a_0) + \delta \sum_{j=1}^{\infty} \beta^j \left(\omega_{12} m(a_j) + \frac{\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}} g(a_j) \right). \end{aligned}$$

We now turn to describe two examples that belong to this weakly separable class. Both of them are widely used in the literature.

Linear production function and CRRA utility function This example can be interpreted as a growth model with a linear production or a consumption-saving problem with a constant interest rate. Without the loss of generality, the resource constraint can be written as $c + k' = \theta k$. Assume the period utility function takes the following CRRA form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and the lifetime utility for an agent in period t is

$$\Psi_t = u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u\left(c_{t+\tau}\right).$$

We use the saving rate as the rescaled action, and show that Assumption 1 is satisfied. Given an initial capital level k_0 and a sequence of saving rate $\{s_\tau\}_{\tau=0}^{\infty}$, the implied sequence of capital is simply $k_t = \Pi_{\tau=0}^{t-1} s_\tau \theta^t k$. The lifetime utility can be rewritten as

$$\begin{aligned} U(k_0, \{s_\tau\}_{\tau=0}^{\infty}) &= \frac{(\theta k_0)^{1-\sigma}}{1-\sigma} \left\{ (1-s_0)^{1-\sigma} + \delta\beta(s_0(1-s_1)\theta)^{1-\sigma} + \delta\beta^2(s_0 s_1(1-s_2)\theta^2)^{1-\sigma} + \dots \right\} \\ &\equiv \frac{(\theta k_0)^{1-\sigma}}{1-\sigma} V(\{s_\tau\}_{\tau=0}^{\infty}), \end{aligned}$$

which is weakly separable. Therefore, we can apply our organizational equilibrium concept in this environment.

Leisure choice Consider now the case where agents also choose the amount of labor to supply. The production function now includes labor as input $f(k_t, \ell_t) = k_t^\alpha \ell_t^{1-\alpha}$, and the resource constraint is $c_t + k_{t+1} = f(k_t, \ell_t)$. Assume the period utility function is $u(c_t, \ell_t) = \log c_t + \frac{(1-\ell_t)^{1-\gamma}}{1-\gamma}$, and the lifetime utility for the agent at period t is given by

$$\Psi_t = u(c_t, \ell_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u\left(c_{t+\tau}, \ell_{t+\tau}\right).$$

We choose saving rate $s \in [0, 1]$ and labor $\ell \in [0, 1]$ as the rescaled action. With initial capital $k_0 = k$, the saving rate sequence $\{s_\tau\}_{\tau=0}^{\infty}$, and the labor sequence $\{\ell_\tau\}_{\tau=0}^{\infty}$. This will imply the sequence of capital as $k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-1-j}} \ell_j^{(1-\alpha)\alpha^{t-1-j}}$. The total payoff is

$$\begin{aligned}
&U(k_0, s_0, s_1, \ldots, \ell_0, \ell_1, \ldots) \\
= &\frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_0 + \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta} \log s_0 + \delta \sum_{j=1}^{\infty} \beta^j \left(\log(1-s_j) + \frac{\alpha\beta}{1-\alpha\beta} \log s_j \right) \\
&+ \frac{(1-\ell_0)^{1-\gamma}}{1-\gamma} + \frac{\delta\alpha(1-\alpha)\beta}{1-\alpha\beta} \log \ell_0 + \delta \sum_{j=1}^{\infty} \beta^j \left(\frac{(1-\ell_j)^{1-\gamma}}{1-\gamma} + \frac{\alpha(1-\alpha)\beta}{1-\alpha\beta} \log \ell_j \right).
\end{aligned}$$

The weakly separable requirement is satisfied by both the saving rates and the leisure choices, and we can apply our equilibrium concept to each choice separately.

4.2 Approximation with Weakly Separable Economies

The assumption of weakly separable utility is quite restrictive and is often not satisfied. In this section, we propose a strategy to study organizational equilibrium for economies where such assumption is not satisfied. Our approach is to look at an economy that is weakly separable and very similar in a particular metric to the original one, and then study organizational equilibrium in this alternative economy. This strategy has a strong tradition in Macroeconomics where little (if anything) is known about recursive equilibrium in distorted economies that do not have a particular functional form. Consequently, the equilibrium is computed for a similar economy in a certain sense (see Kubler and Schmedders (2005) and Kubler (2007) for a discussion).

Consider a quasi-geometric discounting economy with state variable k and action a. Suppose the preference is

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(k_{t+\tau}, a_{t+\tau}\right),$$

and the state variable evolves according to

$$k_{t+1} = F(k_t, a_t).$$

In general, Ψ_t may not be separable between k and the sequence of actions $\{a_{t+\tau}\}_{\tau=0}^{\infty}$. Instead, we consider the log-linearized version of the original problem around a particular point $(\overline{k},\overline{a})$. This approximated economy is

$$\widehat{\Psi}_t = \widehat{u}(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} \, \widehat{u}\left(k_{t+\tau}, a_{t+\tau}\right).$$

such that

$$\begin{aligned}
\widehat{u}(k,a) &= u\left(\overline{k},\overline{a}\right) + \overline{k} u_k\left(\overline{k},\overline{a}\right)\left(\log(k) - \log(\overline{k})\right) + \overline{a} u_a\left(\overline{k},\overline{a}\right)\left(\log(a) - \log(\overline{a})\right) \\
\log(k') &= \log\left(\overline{k}\right) + F_k\left(\overline{k},\overline{a}\right)\left(\log(k) - \log(\overline{k})\right) + \frac{\overline{a} F_a\left(\overline{k},\overline{a}\right)}{\overline{k}}\left(\log(a) - \log(\overline{a})\right)
\end{aligned}$$

This approximated economy belongs to the class of separable economies discussed in Section 4.1, and therefore it is weakly separable. Unlike in a standard log-linearization exercise, the stationary allocation of this economy under the organizational equilibrium is not known ex ante. Denote the converging point of the transition path $\{a_\tau\}_{\tau=0}^{\infty}$ in the organizational equilibrium as a^*. The natural requirements for the selection of $(\overline{k},\overline{a})$ are $\overline{a} = a^*$ and $\overline{k} = F(\overline{k},\overline{a})$. This leads to the following condition that characterizes the steady state $\overline{a}$

$$\left(1 - \beta(1-\delta)\right)\left(1 - \beta F_k\left(\overline{k},\overline{a}\right)\right) u_a\left(\overline{k},\overline{a}\right) + \delta \beta u_k\left(\overline{k},\overline{a}\right) F_a\left(\overline{k},\overline{a}\right) = 0. \tag{4.1}$$

Once $(\overline{k},\overline{a})$ are fixed according to equation (4.1), the entire transition path can be derived in a way similar to Section 2.2

$$a_{t+1} = \exp\left\{\frac{\log(a_t) + \frac{\delta\beta u_k(\overline{k},\overline{a}) F_a(\overline{k},\overline{a})}{u_a(\overline{k},\overline{a})(1-\beta F_k(\overline{k},\overline{a}))}\log(a_t) - (1-\beta(1-\delta))\log(\overline{a})}{\beta(1-\delta)}\right\}. \tag{4.2}$$

The detailed derivation can be found in the Appendix. We will also utilize this approximated economy in the quantitative taxation problem in the next section.

5 Organizational Equilibrium and Public Policy

In this Section, we put the notion of Organizational Equilibrium to work for the cases that we find most interesting, those of the determination of government policies when the Ramsey solution is time inconsistent.

To look at these environments we extend the framework in Section 3 to accommodate a government (a large player) who behaves strategically, and representative households who behave competitively. Given the current level of $k \in K$, the government chooses an action a from a set A, and the consumers choose an action s from the set $s(k) \subseteq S$. The state evolves according to $k' = F(k, x, s)$. Let the preferences for the government in period t be given by $\Psi(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots)$.[6]

Assumption 5. *Given a sequence of government actions $\boldsymbol{a} := \{a_t\}_{t=0}^{\infty}$, there exists a unique competitive equilibrium $\boldsymbol{s(a)} := \{s_t(\boldsymbol{a})\}_{t=0}^{\infty}$, where the sequence $\boldsymbol{s(a)}$ is independent of the state k_0.*

Assumption 5 plays two roles. First, the uniqueness allows us to define government preferences directly over the sequence of government actions, taking as given that households will play the associated competitive equilibrium. Second, the fact that $\mathbf{s}$ is independent of the initial state extends the weak separability requirement that is at the heart of our method. We can then define the government's preferences over sequences of actions as

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) := \Psi(k, a_t, s_t(\mathbf{a}), a_{t+1}, s_{t+1}(\mathbf{a}), a_{t+2}, s_{t+2}(\mathbf{a}), \ldots), \tag{5.1}$$

where for $t \geq 0$, k_t is computed recursively as

$$k_{t+1}(k) = F(k_t(k), x_t, s_t(\mathbf{a})). \tag{5.2}$$

These preferences take now the same form as in the one-agent case, so we impose once again Assump-

[6] As in Section 3, sometimes it may be necessary to transform the original government action so that it is feasible independently of the choices of the private sector and the current level of the physical state, and so that the desired separability property of preferences emerges. A similar rescaling may be needed for the household choices.

tions 1 and 3, and we define an organizational equilibrium as in Definition 1. While the definition of an organizational equilibrium is the same in terms of sequences of actions, its connection to symmetric subgame-perfect equilibria of an underlying game is slightly different, due to the presence of competitive households that act in anticipation of the government's future actions. We describe this game in detail in Appendix D.1. Two aspects are worth pointing out. First, as in the application that we will describe shortly, we assume that the government is a first mover within each period, so that households react contemporaneously to a government deviation.[7] Second, the equilibrium strategies that gradually reward the government from abstaining from short-run temptations and conversely reverse those rewards in the event of a deviation rely on a coordination of the beliefs of the private sector, rather than simply on the actions of future policymakers.

5.1 A Simple Taxation Example

To illustrate the general definition of an organizational equilibrium in a hybrid competitive-strategic environment, we revisit Klein, Krusell, and Ríos-Rull (2008), replacing their Markov equilibrium with our notion of organizational equilibrium. In this problem, the government sets a tax instrument, which, depending on the case, is a flat tax on capital income, labor income, or total income. The proceeds are used to produce a public good, and the government is constrained to a balanced budget. In this subsection, we first consider a special case with inelastic labor supply and full depreciation, where closed-form solution is possible. We then explore the quantitative version as in Klein, Krusell, and Ríos-Rull (2008) in the next subsection.

The production function is given by

$$y_t = f(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha},$$

where labor is inelastically supplied ($\ell_t = 1$) and capital is subject to full depreciation, so that the

[7] Of course, the definition could be adapted to environments where the opposite timing prevails.

resource constraint is

$$c_t + g_t + k_{t+1} = f(k_t, \ell_t). \tag{5.3}$$

g_t is the government provision of the public good. Preferences are

$$\sum_{t=0}^{\infty} \beta^t \, [\log c_t + \gamma \log g_t].$$

We derive the analytical expressions for the case in which the government instrument is a tax on capital income. The same method can be applied when the tax is levied on labor income, or on total income. The consumers' budget constraint is

$$c_t + k_{t+1} = (1 - \tau_t) r_t k_t + w_t.$$

We take the tax rate to be the government's action. Its domain is $[0, 1]$ and is thus independent of initial capital. To avoid dealing with the complications of infinitely negative utility, we constrain the government to choices in $[\epsilon, 1-\epsilon]$, where $\epsilon > 0$ can be chosen arbitrarily small so that the bounds are never hit in the equilibrium we consider. Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$, we first characterize a competitive equilibrium in terms of sequences of consumption, capital, and factor prices, and then summarize it by a sequence of saving rates $s_t \in [0, 1]$, which is our notion of private sector's actions.

Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ and an initial level of capital k_0, a sequence $\{c_t, g_t, k_{t+1}, w_t, r_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if the following conditions are satisfied:

- Factor prices are equal to their marginal productivity, i.e.,

$$r_t = f_k(k_t),$$
$$w_t = f(k_t) - r_t k_t;$$

- The household's intertemporal decision is optimal, which requires the Euler condition to hold

$$u'(c_t) = \beta u'(c_{t+1})(1-\tau_{t+1})r_{t+1},$$

along with the transversality condition

$$\lim_{t\to\infty} \beta^t u'(c_t)k_{t+1} = 0;$$

- The government budget is balanced, i.e.,

$$g_t = \tau_t r_t k_t;$$

- And the resource constraint (5.3) holds.

Substituting factor prices, the resource constraint, and the budget constraint into the Euler equation and summarizing private-sector actions by the saving rage $s_t := k_{t+1}/f(k_t, \ell_t)$ (which also is in $[0,1]$ independently of initial capital), a competitive equilibrium is described by the difference equation

$$\frac{s_t}{1-s_t-\alpha\tau_t} = \frac{\alpha\beta(1-\tau_{t+1})}{1-s_{t+1}-\alpha\tau_{t+1}}. \tag{5.4}$$

along with the transversality condition

$$\lim_{t\to\infty} \beta^t \frac{s_t}{1-\alpha\tau_t - s_t} = 0. \tag{5.5}$$

Lemma 1. *Assumption 5 is satisfied for this economy. Specifically, given a sequence* $\{\tau_t\}_{t=0}^{\infty} \in [\epsilon, 1-\epsilon]^{\infty}$, *there exists a unique competitive equilibrium.*

Suppose the sequence of tax rates is $\{\tau_j\}_{j=0}^{\infty}$, the sequence of saving rates is $\{s_\tau\}_{\tau=0}^{\infty}$, and the initial capital is k_0. Then the sequence of capitals is,

$$k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-1-j}},$$

and the current government's total payoff is

$$U(k_0, s_0, s_1, \ldots, \tau_0, \tau_1, \ldots) = \frac{\gamma}{1-\beta} \log \alpha + \frac{\alpha(1+\gamma)}{1-\alpha\beta} \log k_0 + \sum_{j=0}^{\infty} \beta^j \left\{ \log(1 - \alpha\tau_j - s_j) + \gamma \log \tau_j + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_j \right\}.$$

Clearly, the weakly separable condition is satisfied in this environment.

In an organizational equilibrium, the action payoff should be equalized for governments in different periods, i.e., $\sum_{j=0}^{\infty} \beta^j \left\{ \log(1 - \alpha\tau_{t+j} - s_{t+j}) + \gamma \log \tau_{t+j} + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_{t+j} \right\}$ should equal to a constant for different t. Utilizing the recursive structure, the following lemma provides a system of first-order equations that links the current and future tax rates.

Lemma 2. *In an organizational equilibrium, given the current tax rate τ, the current saving rate s, the future tax rate τ' and s' need to satisfy the following system of equations for some constant $\overline{V}$*

$$(1-\beta)\overline{V} = \log(1 - \alpha\tau - s) + \gamma \log \tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s, \tag{5.6}$$

$$(1-\beta)\overline{V} = \log(1 - \alpha\tau' - s') + \gamma \log \tau' + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s', \tag{5.7}$$

$$\frac{s}{(1 - s - \alpha\tau)} = \frac{\alpha\beta(1-\tau')}{1 - s' - \alpha\tau'}. \tag{5.8}$$

Intuitively, equation (5.6) and (5.7) make sure that the action payoff for the current and future government are the same. Equation (5.8) corresponds to the Euler equation in the private sector. Given τ, there could be two different saving rates s that satisfy equation (5.6). To proceed, define

$g\left(\tau;\overline{V}\right)$ as

$$g\left(\tau;\overline{V}\right)=\min\left\{s\in(0,1)\;\middle|\;\log(1-\alpha\tau-s)+\gamma\log\tau+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s=(1-\beta)\overline{V}\right\} \tag{5.9}$$

which selects the smaller saving rates that delivers the action payoff $\overline{V}$. The system (5.6) to (5.8) is too complicated to allow an analytical solution, and we have verified numerically that there does not exist a solution to the system if the larger saving rate in equation (5.6) is selected. Therefore, in terms of equation (5.7), only $g(\tau';\overline{V})$ can be chosen as well. Otherwise, there will be no solution in the next period.

Similar to Section 2.2, the organizational equilibrium can be characterized by an optimal proposal function that governs the transition, and a starting point that implements the no-delay condition.

Proposition 5. *The sequence of tax rates in the organizational equilibrium can be obtained recursively by the proposal function $q^*(\tau)$ which satisfies*

$$\frac{g(\tau;V^*)}{1-g(\tau;V^*)-\alpha\tau}=\frac{\alpha\beta(1-q^*(\tau))}{1-g(q^*(\tau);V^*)-\alpha q^*(\tau)}, \tag{5.10}$$

where V^* is defined as

$$(1-\beta)V^*=\max_{\tau}\;\log(1-\alpha\tau-\alpha\beta(1-\tau))+\gamma\log\tau+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log(\alpha\beta(1-\tau)). \tag{5.11}$$

The initial tax rate τ_0 satisfied $\Phi(\tau_0)\leq V^*$, where $\Phi(\tau_0)$ is given by

$$\Phi(\tau_0)=\max_{\tau_{-1}}\;\log(1-\alpha\tau_{-1}-s_{-1})+\gamma\log\tau_{-1}+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_{-1}+\beta V^* \tag{5.12}$$

$$s.t.\qquad \frac{s_{-1}}{1-s_{-1}-\alpha\tau_{-1}}=\frac{\alpha\beta(1-\tau_0)}{1-g(\tau_0;V^*)-\alpha\tau_0} \tag{5.13}$$

To understand this result better, we again make comparisons with the Markov equilibrium and the Ramsey outcome. Define the following object which can be interpreted as the steady state action payoff

$$\mathcal{H}(s,\tau) \equiv \log(1-\alpha\tau-s) + \gamma\log\tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s.$$

In the Markov equilibrium, the problem of the current government is

$$\max_{\tau} \quad \mathcal{H}(s,\tau), \qquad \text{s.t.} \qquad \frac{s}{(1-s-\alpha\tau)} = \frac{\alpha\beta(1-\tau')}{1-s'-\alpha\tau'}.$$

The current government takes future τ' and s' as given when choosing τ. It follows that the tax rate in the Markov equilibrium is a constant $\tau^M = \frac{\gamma(1-\alpha\beta)}{\alpha(1+\gamma)}$.[8] A high tax rate is chosen because the government fails to take into account the effects of the current tax rate on past saving choices.

In the steady state of the Ramsey outcome, the allocation is characterized by

$$\mathcal{H}_s = \frac{1}{\alpha}\mathcal{H}_\tau, \qquad \text{and} \qquad s = \alpha\beta(1-\tau).$$

This allocation does not maximize $\mathcal{H}(s,\tau)$, but it is optimal from the initial government's perspective. The effect of a high tax rate on past saving decisions is internalized and a low tax rate is set in the long run.

In the organizational equilibrium, the steady-state tax rate maximizes the steady-state action payoff

$$\max_{\tau} \quad \mathcal{H}(s,\tau) \qquad \text{s.t.} \qquad s = \alpha\beta(1-\tau).$$

Unlike the Markov equilibrium, the intertemporal distortion is internalized. This is why the outcome in the organizational equilibrium is better than that in the Markov equilibrium. However, the allocation does not necessarily favor the initial agent and has to strike a balance across governments in different periods, which prevents it being the same as the Ramsey outcome.

As a numerical example, we set $\beta = 0.9, \alpha = 0.36$, and $\gamma = 0.5$, and the proposal function is plotted

[8] The same result is obtained in Klein, Krusell, and Ríos-Rull (2008) where the Markov equilibrium is obtained by taking the limit of the solution to a finite-horizon problem.

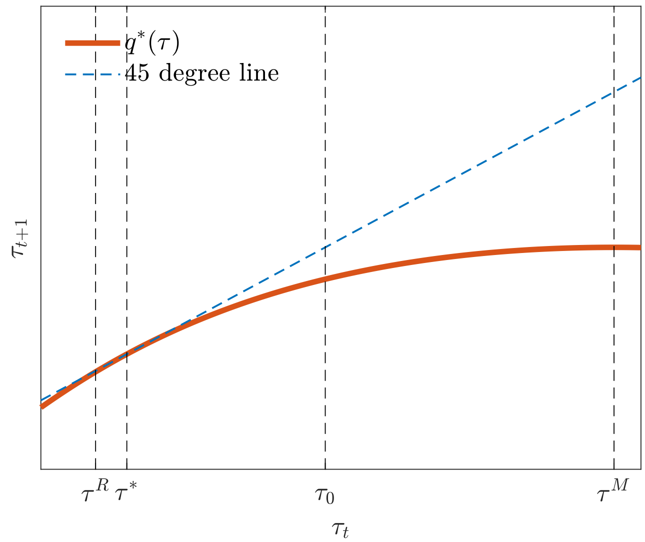
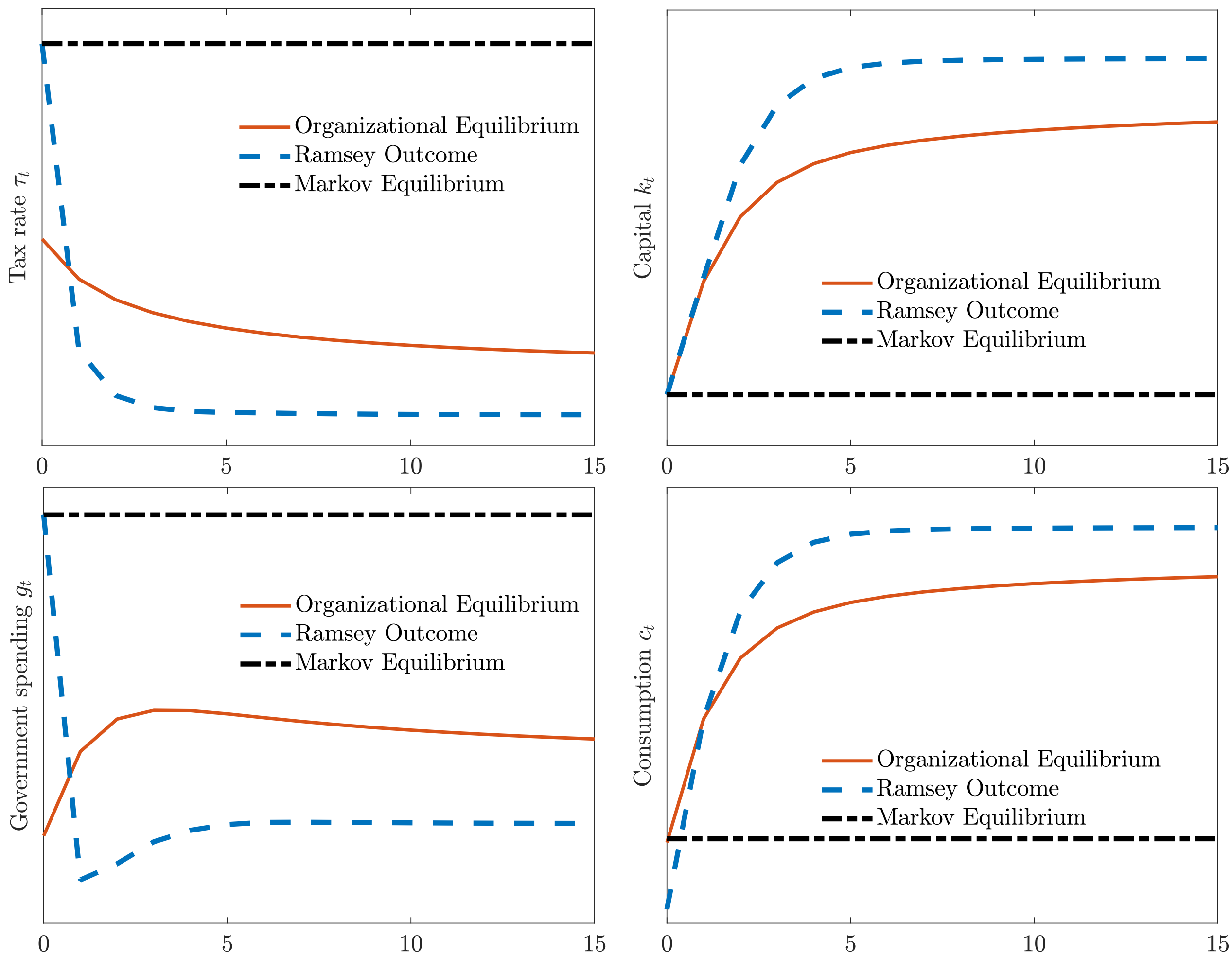


FIGURE 5: Proposal Function $q^*(\tau)$

in Figure 5. Let τ^M denotes the tax rate in the Markov equilibrium and τ^R denote the steady-state tax rate in the Ramsey outcome. As expected, the initial tax rate τ_0 is lower than τ^M, but it is higher than τ^R. The proposal function implies a gradual transition of the tax rate from τ_0 to the steady state τ^*, which is close to that in the Ramsey outcome.

Figure 6 displays the corresponding transition paths for the tax rates and allocation in the three economies. The initial capital is chosen to be the steady state capital level in the Markov economy. In the Ramsey outcome, the government initially sets the tax rate as high as in the Markov equilibrium, and rapidly adjusts it to the steady state value τ^R. As a result, the private consumption drops initially since households anticipating a lower tax rate in the future. At the same time, the capital stock accumulates to its steady state high level. The path of government spending is non-monotonic, since the output and tax rate move in the opposite direction. In the organizational equilibrium, the tax rates starts lower than the Markov tax rate, and it converges to τ^* which is still higher than the long-run level in the Ramsey outcome. The transition of the tax rate is slower than the Ramsey outcome to make sure that the government action payoff is equalized across periods. Because of a lower tax rate, the capital stock is higher than the Markov equilibrium.

Figure 6: Transition Path



In equilibrium, the government in each period will obtain the same constant action payoff V^*. However, the no-delay condition prevents a constant tax rate, and the sequence of tax rates only approach to its steady state level τ^* gradually. If the initial tax rate τ_0 is known, then the entire transition path can be computed recursively via the proposal function $q^*(\tau)$. The condition that $\Phi(\tau_0) \leq V^*$ then guarantees that the initial government has no incentive to wait for the next government to make the equilibrium proposal. As in Section 2.2, we will select τ_0 such that $\Phi(\tau_0) = V^*$ which yields the highest total payoff for the next government.

5.2 A Quantitative Taxation Model

In this section, we revisit the quantitative taxation problem in Klein, Krusell, and Ríos-Rull (2008). We extend previous section to allow elastic labor supply, partial depreciation, and three types of taxation. The preference is

$$\sum_{t=0}^{\infty} \beta^t \left[\gamma_c \log c_t + \gamma_\ell \log(1 - \ell_t) + \gamma_g \log g_t\right].$$

where ℓ_t stands for labor. The budget constraint for the household is

$$c_t + i_t = w_t \ell_t + r_t k_t - (\tau_t^\ell + \tau_t) w_t \ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k + \tau_t)}{r_t}\right) r_t k_t$$

where i_t is investment, τ_t^ℓ is labor income tax, τ_t^k is capital income tax, and τ_t is total income tax. The last term on the right-hand of the budget constraint allows for the possibility of capital depreciation deduction. In Klein, Krusell, and Ríos-Rull (2008), the capital evolves according to

$$k_{t+1} = (1 - \delta) k_t + i_t.$$

However, this specification does not allow the government's lifetime utility to be separable between capital and the sequence of tax rates. Instead, we specify the law of motion of capital to be

$$\log k_{t+1} = \log \overline{k} + (1 - \delta) \log k_t + \delta \log i_t.$$

This can be viewed as a log-linear approximation of the original law of motion, and it fits into the class of separable economies discussed in Section 4.1. This modification delivers the weakly separable property, and therefore we can apply the organizational equilibrium to this approximated economy.

We set the parameters to be $\alpha = 0.36, \beta = 0.96, \delta = 0.08, \gamma_g = 0.09, \gamma_c = 0.27, \gamma_\ell = 0.64$. We choose γ_g such that the steady state government spending to GDP ratio is 18% in the Pareto efficient allocation, and γ_ℓ such that the working time is 35% of a day. The rest of the parameters are

Table 1: Steady State Comparison

Aggregate statistics	Labor income tax				Capital income tax				Total income tax			
	Pareto	Ramsey	Markov	Organization	Pareto	Ramsey	Markov	Organization	Pareto	Ramsey	Markov	Organization
y	1.000	0.701	0.711	0.706	1.000	0.588	0.347	0.553	1.000	0.669	0.679	0.674
k/y	2.959	2.959	2.959	2.959	2.959	1.735	0.624	1.529	2.959	2.528	2.579	2.553
c/y	0.510	0.510	0.544	0.527	0.510	0.712	0.666	0.704	0.510	0.533	0.555	0.544
g/y	0.254	0.254	0.219	0.236	0.254	0.149	0.284	0.174	0.254	0.265	0.239	0.252
c/g	2.008	2.008	2.483	2.234	2.008	4.785	2.345	4.045	2.008	2.008	2.325	2.156
ℓ	0.350	0.245	0.249	0.247	0.350	0.278	0.292	0.280	0.350	0.256	0.257	0.256
τ		0.397	0.342	0.369		0.673	0.916	0.732		0.332	0.301	0.317

standard in the literature. The properties of the transition path are very similar to what we have shown in the previous section, and in this subsection focus on their steady-state properties.

Table 1 shows the steady-state comparison among the Pareto allocation, the Ramsey outcome, the Markov equilibrium, and the organizational equilibrium. The results regarding the Ramsey outcome and the Markov equilibrium are very similar to those obtained in Klein, Krusell, and Ríos-Rull (2008). Throughout the three different tax instruments, a common feature is that the tax rates and the allocation in the organizational equilibrium always stay between the Ramsey outcome and the Markov equilibrium. This feature reinforces the analysis in Section 2.2 and Section 5.1, that the the temptation to favor the current generation is limited in the organizational equilibrium. The capital income taxation is the most distortionary one among the three taxes. In the Markov equilibrium, the output level is only 35% compared with the Pareto efficient output level, while the output level in the organizational equilibrium is 55% of the Pareto allocation and it is only slightly below the Ramsey outcome. We interpret this result as a large improvement over the Markov equilibrium. For labor income tax and total income tax, the difference between the Ramsey outcome and the Markov equilibrium is much smaller. Since the organizational equilibrium stays in between of the two benchmarks, we only conclude that it brings the allocation closer to the Ramsey outcome.

6 Conclusion

TO BE ADDED.

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Appendix

A Proof of Proposition 1

Proof. Suppose the initial agent with capital $k_0 = k$ proposes a sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$, which yields a sequence of capital $\{k_\tau\}_{\tau=0}^{\infty}$. Consider a subsequence of the proposed saving rates from time t on, $\{s_\tau\}_{\tau=t}^{\infty}$. This is the sequence of saving rates that will be used by the agent with capital k_t. The lifetime utility for the agent with capital k_t is

$$U_t(k_t, \{s_\tau\}_{\tau=t}^{\infty}) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + V_t,$$

where

$$V_t = \log(1-s_t) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_t) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_{t+j}) + \frac{\alpha\beta}{1-\alpha\beta}\log(s_{t+j})\right). \tag{A.1}$$

The link between V_t and V_{t+1} is given by

$$V_t - \beta V_{t+1} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - \beta(1-\delta)\log(1-s_{t+1}). \tag{A.2}$$

We will proceed by guessing and verifying. We first ignore condition 2 in the definition of organizationally admissible sequence, which will imply that a proposal has to be that $V_t = \overline{V}$. This is because V_t has to be weakly increasing, and the proposer can be better off by copying future agents' saving rate sequence if $V_t > V_0$ for some t. Later on, we will verify that condition 2 can be satisfied by a particular selection of a saving rate sequence.

The constant action payoff restriction, $V_t = \overline{V}$, simplifies equation (A.2) to

$$(1-\beta)\overline{V} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - \beta(1-\delta)\log(1-s_{t+1}). \tag{A.3}$$

Equation (A.3) has two implications. First, it establishes a recursive relationship for saving rates between two consecutive agents. Second, it provides us a way to select the saving rates sequence that yields the highest utility.

Denote by $s = q(s^-;\overline{V})$ as the proposal function that specifies the current generation's saving rate as a function of last generation's saving rate:

$$q(s^-;\overline{V}) = 1 - \exp\left\{\frac{-(1-\beta)\overline{V} + \frac{\delta\alpha\beta}{1-\alpha\beta}\log s^- + \log(1-s^-)}{\beta(1-\delta)}\right\}. \tag{A.4}$$

If the initial agent wants to make a proposal, it will choose the highest $\overline{V}$ possible. Given a particular $\overline{V}$, the fixed point of the function $q(s^-;\overline{V})$ solves

$$(1-\beta)\overline{V} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) - \beta(1-\delta)\log(1-s). \tag{A.5}$$

Also note that

$$\frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) - \beta(1-\delta)\log(1-s) \in (-\infty, V^*],$$

where V^* is given by

$$s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta}, \tag{A.6}$$

$$V^* = \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s^*) + \frac{\alpha\delta\beta}{(1-\beta)(1-\alpha\beta)}\log s^*. \tag{A.7}$$

If the initial agent chooses $\overline{V} > V^*$, then there is no fixed point for $q(s^-;\overline{V})$. Meanwhile, $q(s^-;\overline{V}) > s^-$ when $\overline{V} > V^*$, and s will converge to 1 at a rate which will make the sum in equation (A.1) diverge to $-\infty$, which cannot be optimal. Therefore, the optimal choice is V^*. The optimal proposal function associated with V^* is $q^*(s^-) = q(s^-;V^*)$.

If we ignore condition 2, there are many valid proposals that the initial agent could make; as an example, a constant saving rate s^* would be one of them. However, this constant saving rate sequence $\{s^*\}$ will violate condition 2. If the initial agent waits for the next generation to propose this constant saving rate sequence, then the initial agent can choose the Markov saving rate $\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}$, which yields a higher utility. This will be the case for all proposals in which

$$s_0 \in \left(q^*\left(\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}\right), s^*\right].$$

Furthermore, simple but tedious algebra shows that $ds/ds^- = 1$ and $d^2s/d(s^-)^2 > 0$ at $s = s^*$ when $\overline{V} = V^*$. s^* is thus a semi-stable steady state: it is stable from the left, but unstable from the right. Proposals in which $s_0 > s^*$ would lead the difference equation to converge to 1, which is ruled out by the same argument which we used to establish that values above V^* are unattainable.

Equation (A.4) attains a minimum when s^- is at the Markov saving rate and is strictly decreasing below this value, converging to 1 as s^- converges to 0. Hence, there exists a value $\underline{s}$ such that $q^*(s) > s^*$ for $s < \underline{s}$: this is a lower value on the initial saving rate, since once again the sequence would otherwise yield arbitrarily negative utility. This $\underline{s}$ also satisfies $q^*(\underline{s}) = s^*$ and is given by

$$\underline{s} = \min\left\{s : \frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) = (1-\beta)V^* + \beta(1-\delta)\log(1-s^*)\right\}$$

The set of organizational equilibria is given by the set of sequences which satisfy (2.9), (2.10), and (A.4), and which have

$$s_0 \in \left[\underline{s}, q^*\left(\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}\right)\right].$$

□

B Proof of Proposition 3.

To prove this we rely on a useful lemma, which introduces a convenient way of representing equilibria through their values, similarly to Abreu, Pierce, and Stacchetti's (1986; 1990) method.[9]

Lemma 3. *Let $V^* \in \mathbb{R}$ and $\hat{\mathcal{V}} \subset \mathbb{R}$ be a value and a set of continuation values that satisfy the following properties:*

1. $$\forall a \in A \quad \exists \hat{v} \in \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) \leq V^*;$$

2. $$\forall v \in \hat{\mathcal{V}} \quad \exists (a, \hat{v}) \in A \times \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) = V^* \wedge W(a, \hat{v}) = v$$

3. *There exists no value $V^{**} > V^*$ and set $\hat{\hat{\mathcal{V}}}$ that satisfies properties 1 and 2; furthermore, there is no set $\hat{\mathcal{V}}_a \supset \hat{\mathcal{V}}$ that satisfies properties 1 and 2 together with V^*.*

[9]Note, however, that we cannot adopt their method to recursively compute the desired sets. *Given* V^*, $\hat{\mathcal{V}}$ can be computed recursively as in Abreu, Pierce, and Stacchetti. However, without further assumptions the set of values of V^* for which $\hat{\mathcal{V}}$ is defined need not be convex, which makes finding its maximum difficult.

Then:

- *Construct an arbitrary sequence of actions* $\{a_t^*\}_{t=0}^{\infty}$ *recursively as follows. In period 0, pick* $\hat{v}_0^* \in \hat{\mathcal{V}}$ *and* $(a_0^*, \hat{v}_1^*) \in A \times \hat{\mathcal{V}}$ *such that* $\tilde{V}(a_0^*, \hat{v}_1^*) = V^*$ *and* $W(a_0^*, \hat{v}_1^*) = \hat{v}_0^*$*. In each subsequent period, pick* $(a_t^*, \hat{v}_{t+1}^*) \in A \times \hat{\mathcal{V}}$ *such that* $\tilde{V}(a_t^*, \hat{v}_{t+1}^*) = V^*$ *and* $W(a_t^*, \hat{v}_{t+1}^*) = \hat{v}_t^*$*. Constructing such a sequence is possible by the definition of* V^* *and* $\hat{\mathcal{V}}$*. The sequence so constructed is the outcome of a reconsideration-proof equilibrium;*
- *If* $\{a_t^*\}_{t=0}^{\infty}$ *is the equilibrium path of a reconsideration-proof equilibrium,* $\tilde{V}(a_0^*, a_1^*, ...) = V^*$ *and* $\hat{V}(a_t^*, a_{t+1}^*, ...) \in \hat{\mathcal{V}}$ *for any* $t > 0$*.*

Proof.

First, we prove that the recursively-constructed sequence $\{a_t^*\}_{t=0}^{\infty}$ satisfies

$$\tilde{V}(a_t^*, \hat{V}(a_{t+1}^*, a_{t+2}^*, ...)) = V^* \quad \forall t \geq 0 \tag{B.1}$$

and

$$\widehat{V}(a_t^*, a_{t+1}^*, a_{t+2}^*, ...) \in \hat{\mathcal{V}} \quad \forall t \geq 0. \tag{B.2}$$

Note that, if $\widehat{v}_T^* = \widehat{V}(a_T^*, a_{T+1}^*, a_{T+2}^*, ...)$ for some period T, iterating backwards we find that $\widehat{v}_t^* = \widehat{V}(a_t^*, a_{t+1}^*, a_{t+2}^*, ...)$ for all $t < T$, so that equations (B.1) and (B.2) hold.

Define

$$\{\underline{a}_t\}_{t=0}^{\infty} \in \arg\min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)$$

and similarly let $\{\bar{a}_t\}_{t=0}^{\infty}$ be a sequence that attains the maximum. Both exist by the compactness of A and the continuity of $\widehat{V}$ (in the product topology).

Next, truncate the sequence $\{a_t^*\}_{t=0}^{\infty}$ at time $S > T$ and replace the continuation with $\{\underline{a}_t\}_{t=0}^{\infty}$ or $\{\bar{a}_t\}_{t=0}^{\infty}$. By assumption 4 and the monotonicity of W, we have

$$\widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \underline{a}_0, \underline{a}_1, ...) \leq \widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, a_{S+1}^*, a_{S+2}^*, ...) \leq \widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \bar{a}_0, \bar{a}_1, ...) \tag{B.3}$$

and

$$\widehat{V}(a^*_T, a^*_{T+1}, ..., a^*_S, \underline{a}_0, \underline{a}_1, ...) = W(a^*_T, W(a^*_{T+1}, ...W(a^*_S, W(\underline{a}_0, W(\underline{a}_1, ...)...))...)) \leq$$
$$W(a^*_T, W(a^*_{T+1}, ...W(a^*_S, \hat{v}^*_S)...)) = \hat{v}^*_T \leq \tag{B.4}$$
$$W(a^*_T, W(a^*_{T+1}, ...W(a^*_S, W(\bar{a}_0, W(\bar{a}_1, ...)...))...)) = \widehat{V}(a^*_T, a^*_{T+1}, ..., a^*_S, \bar{a}_0, \bar{a}_1, ...)$$

Taking limits as $T \to \infty$ in equations (B.3) and (B.4) and exploiting the continuity of $\widehat{V}$ according to the product topology, the left-most and right-most expressions in the inequalities converge to the same value, which then implies that indeed $\widehat{v}^*_T = \widehat{V}(a^*_{T+1}, a^*_{T+2}, a^*_{T+3}, ...)$ and (B.1) and (B.2) hold.

To complete the proof of the first point, we need to show that there exists no symmetric subgame-perfect equilibrium whose payoff is strictly greater than V^*. By contradiction, suppose that there is such an equilibrium with value $V^{**} > V^*$. Let σ^{**} be the strategy profile representing one such equilibrium. Define

$$\hat{\mathcal{V}}_b := \{v : v = \widehat{V}(a^{**}_{t+1}|_{h^t}, a^{**}_{t+2}|_{h^t}, a^{**}_{t+3}|_{h^t}, ...),\ h^t \in A^t\},$$

where $\{a^{**}_s|_{h^t}\}^{\infty}_{s=t+1}$ is the equilibrium path implied by the strategy profile σ^{**} following a history h^t. The pair $(V^{**}, \hat{\mathcal{V}}_b)$ satisfies property 1 in the lemma, since otherwise σ^{**}_0 would not be optimal at time 0. It also satisfies property 2 since σ^{**} is symmetric and by the definition of $\hat{\mathcal{V}}_b$. But then this implies that property 3 in the lemma does not hold for V^*, establishing a contradiction.

In the previous point we proved that, given V^* and $\hat{\mathcal{V}}$, we can construct a reconsideration-proof equilibrium of value V^*. Since all reconsideration-proof equilibria must have the same value, it must be the case that $\tilde{V}(a^*_0, a^*_1, ...) = V^*$. Furthermore, repeating the steps of the previous point, we can prove that the value V^* and the set

$$\hat{\mathcal{V}}_a := \{v : v = \widehat{V}(a^*_{t+1}|_{h^t}, a^*_{t+2}|_{h^t}, a^*_{t+3}|_{h^t}, ...),\ h^t \in A^t\},$$

satisfy properties 1 and 2. By the definition of $\hat{\mathcal{V}}$, it follows that $\hat{\mathcal{V}}_a \subseteq \hat{\mathcal{V}}$. □

While not essential for the proof of Proposition 3, the following lemma is useful for computations:

Lemma 4. *The set $\hat{\mathcal{V}}$ defined in Lemma 3 is convex.*[10]

[10] Lemma 3 defines a unique set, since the union of all sets satisfying properties 1 and 2 satisfies properties 1 and 2 as well.

Proof. We first define the set $\hat{\mathcal{V}}_c$ by relaxing property 2 in Lemma 3 to be the following:

$$\forall v \in \hat{\mathcal{V}}_c \quad \exists (a, \hat{v}) \in A \times \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) \geq V^* \wedge W(a, \hat{v}) = v. \tag{B.5}$$

We will later prove that $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$.

Simple case. First, if $\hat{\mathcal{V}}_c$ is a singleton, then it is necessarily convex and $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$: by property 3 of Lemma 3, V^* should be raised until $\tilde{V}(a, \hat{v}) = V^*$ at the single element $\hat{v} \in \hat{\mathcal{V}}_c$, with no effect on property 2 and relaxing the constraint in property 1.

From now on, we study the case in which $\hat{\mathcal{V}}_c$ contains at least two values.

Step 1. To prove that $\hat{\mathcal{V}}_c$ is convex, we prove that its convex hull, $\mathrm{Co}(\hat{\mathcal{V}}_c)$, satisfies properties 1 and 2 as well (and of course $\mathrm{Co}(\hat{\mathcal{V}}_c) \supset \hat{\mathcal{V}}_c$ unless $\hat{\mathcal{V}}_c$ is convex as well). Property 1 is immediate from the monotonicity of $\tilde{V}$. Let $v_1, v_2 \in \hat{\mathcal{V}}_c$, and let $(a_1, \hat{v}_1)$, $(a_2, \hat{v}_2)$ elements of $A \times \hat{\mathcal{V}}_c$ be two pairs of actions and continuation values that satisfy property 2 of Lemma 3. Consider their convex combination $(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2)$, $\alpha \in [0, 1]$. Since $\tilde{V}$ is continuous and quasiconcave and W is continuous, $\tilde{V}(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2) \geq V^*$, and $W(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2)$ takes all values in $[v_1, v_2]$ as α varies between 0 and 1. Hence, all intermediate values satisfy property 2 as well, which completes the proof that $\mathrm{Co}(\hat{\mathcal{V}}_c)$ satisfies property 2.

Step 2. To prove that $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$, proceed as follows. Define $\underline{v}_c := \min\{\hat{\mathcal{V}}_c\}$ and $\bar{v}_c := \max\{\hat{\mathcal{V}}_c\}$.[11] By definition, we can find $(\underline{a}, \hat{\underline{v}})$ and $(\bar{a}, \bar{\hat{v}})$ such that

$$\tilde{V}(\underline{a}, \hat{\underline{v}}) \geq V^* \wedge W(\underline{a}, \hat{\underline{v}}) = \underline{v}$$

and

$$\tilde{V}(\bar{a}, \bar{\hat{v}}) \geq V^* \wedge W(\bar{a}, \bar{\hat{v}}) = \bar{v}.$$

Since A is convex, we can construct within it a line from $\underline{a}$ to $\bar{a}$ by defining $a(\alpha) := \alpha \underline{a} + (1-\alpha)\bar{a}$, $\alpha \in [0, 1]$. By the quasiconcavity of $\tilde{V}$, we know

$$\tilde{V}(a(\alpha), \alpha \hat{\underline{v}} + (1-\alpha)\bar{\hat{v}}) \geq V^*.$$

[11] It is straightforward to prove that $\hat{\mathcal{V}}_c$ is closed, by the continuity of the functions defining it.

By property 1 of Lemma 3, for each action $a(\alpha)$ and the monotonicity and continuity of $\tilde{V}$ we have

$$\tilde{V}(a(\alpha), \underline{v}) \leq V^*.$$

Since $\hat{\mathcal{V}}_c$ is convex, we can find a (unique) value $\hat{v}(\alpha)$ such that

$$\tilde{V}(a(\alpha), \hat{v}(\alpha)) = V^*.$$

Monotonicity and continuity of $\tilde{V}$ imply that $\hat{v}(\alpha)$ is a continuous function. It then follows that $\hat{V}(a(\alpha), \hat{v}(\alpha))$ is a continuous function of α. As $\alpha \in [0,1]$, this function must take all values between $\underline{v}$ and $\bar{v}$, proving that the property 2 of Lemma 3 is satisfied by $\hat{\mathcal{V}}_c$ and thus $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$. □

We are now ready to prove Proposition 3.

Proof. The second property of the value V^* and the set $\hat{V}$ in Lemma 3 implies that we can construct a function $g : \hat{\mathcal{V}} \to \mathbb{R} \times \hat{\mathcal{V}}$ with the property that $\tilde{V}(g(v)) = V^*$ and $W(g(v)) = v$.[12] Starting from any value $v_0 \in \hat{\mathcal{V}}$, we can construct recursively a path $(a_t, v_{t+1}) = g(v_t)$. By Lemma 3, this is the equilibrium path of a reconsideration-proof equilibrium. It will thus be an organizational equilibrium provided that

$$V(a_t, v_{t+1}) \geq \max_a \tilde{V}(a, v_0) \quad \forall t.$$

By the definition of $\mathcal{V}$, this property is satisfied by its least element, $\underline{v}$;[13] hence, it will be satisfied provided that the initial value v_0 is sufficiently low. □

We now proceed to prove Proposition 4.

Proof. Define a correspondence $\zeta : \mathbb{R} \times \mathbb{R} \rightrightarrows \mathbb{R}$ as follows:

$$v \in \zeta(v', v^*) \Longleftrightarrow \exists a \in A : \begin{cases} \tilde{V}(a, v') = v^* \\ W(a, v') = v. \end{cases} \tag{B.6}$$

[12] This function may not be unique. That's true, but I do not think I used uniqueness, or did I miss it somewhere?

[13] By the monotonicity of $\widetilde{V}$ in its second argument and the property 1 of $\mathcal{V}$, $\tilde{V}(a, \underline{v}) \leq V^*$ for all $a \in A$.

In words, given (v^*, v'), v belongs to the correspondence if there is an action a which, together with a continuation value v', yields utility v^* when evaluated according to the decision maker's preferences $(\tilde{V})$ and utility v when evaluated with her continuation utility function W.

We prove that there exists a value v^* for which ζ is nonempty and admits a fixed point in continuation utilities $(v = v')$. We do so by proving that a Markov equilibium (a^M, v^M) exists, such that

$$v^* = \tilde{V}(a^M, v^M) = \max_a \tilde{V}(a, v^M) \tag{B.7}$$

and

$$v^M = W(a^M, v^M). \tag{B.8}$$

To prove the existence of a Markov equilibrium, we construct a correspondence $\hat{a}(.)$ from A into itself by setting

$$\hat{a}(a) = \max_{a_0 \in A} \widehat{V}(a_0, a, a, a, ...).$$

By the usual compactness and continuity properties, this correspondence is nonempty, compact-valued, and upper hemicontinuous. Quasiconcavity of $\widehat{V}$ ensures that it is also convex-valued. Hence, the correspondence has a fixed point by Kakutani's theorem; let a^M be one such fixed point. Given Assumption 4, letting $v^M := \widehat{V}(a^M, a^M, a^M, ...)$, equations (B.7) and (B.8) are satisfied.

We thus know $v^M \in \zeta(v^M, \tilde{V}(a^M, v^M))$. Once again, our assumptions about compactness and continuity imply that the correspondence ζ is upper hemicontinuous. Let V^* be the maximal value for which ζ admits a fixed point in continuation utilities. In the proofs below, it is useful to establish that

$$v \in \zeta(v', V^*) \Longrightarrow v \leq v'. \tag{B.9}$$

Suppose (B.9) is not satisfied. Let (a, v') be such that $V(a, v') = V^*$ and $W(a, v') > v'$. Holding the action a fixed, continuity and monotonicity imply that higher values of v' lead to higher values of $V(a, v')$ and $W(a, v')$. As long as $W(a, v') > v'$, we know that $v' < \max_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)$ and can thus be raised further. Eventually, we will attain a value $v^h > v'$ for which $W(a, v^h) = v^h$ (this has to happen, since $W(a, v')$ is bounded by the maximum above). Let $V^h := V(a, v^h) > V^*$. We just established that a fixed point of $\zeta(., V^h)$ exists, which contradicts the assumption that V^* is the highest value for which a fixed point can be found.

In our next step, we prove that there are no symmetric equilibria with value $V^{**} > V^*$. By the definition of V^*, given any combination of an action and a continuation utility (a, v'), if $\tilde{V}(a, v') = V^{**}$ then $W(a, v') < v'$. This implies that any equilibrium path with value V^{**} would feature a strictly increasing sequence of continuation values; convergence is ruled out, because continuity and compactness would imply that the limiting point would be a fixed point of ζ, which is inconsistent with $V^{**} > V^*$. Since the set of possible continuation values is bound by

$$\max_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, \ldots),$$

no such equilibrium path can exist.

We now prove that there exist symmetric equilibria with value V^*, which then implies that any such equilibrium is reconsideration proof. Let v^{SS} be the maximal fixed point of $\zeta(., V^*)$. For any continuation value $v > v^{SS}$, a repetition of the arguments described above for V^{**} imply that no equilibrium path would be possible.[14] We prove instead that there exists a convex set $\mathcal{V} = [v_\ell, v^{SS}]$ which, together with V^*, satisfies the properties of Lemma 3, where

$$v_\ell := \min_{v' \leq v^{SS}} \min \zeta(v', V^*). \tag{B.10}$$

To do so, prove first that, for any action $a \in A$, $\tilde{V}(a, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) \leq V^*$. By contradiction, suppose that an action a_L such that $\tilde{V}(a_L, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) > V^*$ existed. We could then repeat the same steps used to prove (B.9) and construct a steady state with value higher than V^*.

Since $\tilde{V}(a, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) \leq V^* \quad \forall a \in A$, we can define

$$v'_{\min} := \min_{(a,v')} v' := \tilde{V}(a, v') = V^*.$$

Since there exists an action a^{SS} such that $V(a^{SS}, v^{SS}) = V^*$, $v'_{\min} \leq v^{SS}$. Also, by equations (B.9) and (B.10), $v_\ell \leq v'_{\min}$. Hence, $\tilde{V}(a, v_\ell) \leq V^* \quad \forall a \in A$: Property 1 of Lemma 3 is satisfied by the value V^* and the continuation set $[v_\ell, v^{SS}]$. To prove Property 2, let a_ℓ and v'_ℓ be such that $W(a_\ell, v'_\ell) = v_\ell$ and $\tilde{V}(a_\ell, v'_\ell) = V^*$, and $\lambda \in [0, 1]$.[15] As we just established, $\tilde{V}(\lambda a_\ell + (1-\lambda)a^{SS}, v_\ell) \leq V^*$. By quasiconcavity, $\tilde{V}(\lambda a_\ell + (1 - \lambda)a^{SS}, \lambda v'_\ell + (1-\lambda)v^{SS}) \geq V^*$. Strict monotonicity implies that there exists a unique value v_λ such that $\tilde{V}(\lambda a^{SS} + (1-\lambda)a_\ell, v_\lambda) = V^*$, which must vary continuously with λ by the continuity of $\tilde{V}$. It follows that

[14] If along the equilibrium path, for some $T \geq 0$, $v_T > v^{SS}$, then $v_t > v^{SS}$ for all $t > T$. Since $\{v_t\}$ is bounded and monotonically increasing, the limiting point will be a fixed point of ζ, which is a contradiction to that v^{SS} is the largest fixed point.

[15] We have $v_\ell \leq v'_\ell \leq v^{SS}$ by (B.9) and (B.10).

$W(\lambda a^{SS}+(1-\lambda)a_\ell, v_\lambda)$ is a continuous function of λ and it takes all values between v_ℓ and v^{SS}, proving that Property 2 of Lemma 3 holds. Finally, from equations (B.9) and (B.10), we know that any value $v \notin [v_\ell, v^{SS}]$ could only be attained by some action a with a continuation value $v' > v^{SS}$, which would lead to nonexistence in subsequent periods. Hence, $[v_\ell, v^{SS}]$ is the largest set that satisfies Properties 1 and 2 of Lemma 3 together with the value V^*, completing the proof that a reconsideration-proof equilibrium has value V^*, and thus that in turn the organizational equilibrium with the state variable is also associated with an action value V^*.

Finally, suppose that V is strictly quasiconcave. Let a^{SS} be the unique action that attains $\max_a V(a,a,a,...)$. If this steady state is not a Markov equilibrium, then $a^{SS} < \max_a \tilde{V}(a, v^{SS})$. In this case, a sequence that starts at a^{SS} and stays constant violates the no-delay condition. □

C Approximated Equilibrium

Compared with the class of economies specified in Section 4.1, the approximated equilibrium in Section 4.2 chooses particular function forms for $h(\cdot)$, $m(\cdot)$, and $g(\cdot)$. Here, we derive the general result that also applies to Section 4.2.

The problem can be written as

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u\left(k_{t+\tau}, a_{t+\tau}\right).$$

such that

$$u(k,a) = \omega_{10} + \omega_{11} h(k) + \omega_{12} m(a)$$
$$h(k') = \omega_{20} + \omega_{21} h(k) + \omega_{22} g(a)$$

In the approximated equilibrium, $\boldsymbol{\omega}$ is a constant matrix chosen to match the level and first order derivatives.

$$\boldsymbol{\omega} = \begin{bmatrix} u(\overline{k},\overline{a}) - \frac{u_k(\overline{k},\overline{a})}{h_k(\overline{k})} h(\overline{k}) - \frac{u_a(\overline{k},\overline{a})}{m_a(\overline{a})} m(\overline{a}) & \frac{u_k(\overline{k},\overline{a})}{h_k(\overline{k})} & \frac{u_a(\overline{k},\overline{a})}{m_a(\overline{a})} \\ h(\overline{k}) - F_k(\overline{k},\overline{a}) h(\overline{k}) - \frac{h_k(\overline{k}) F_a(\overline{k},\overline{a})}{g_a(\overline{a})} g(\overline{a}) & F_k(\overline{k},\overline{a}) & \frac{h_k(\overline{k}) F_a(\overline{k},\overline{a})}{g_a(\overline{a})}. \end{bmatrix}$$

Given a sequence of rescaled actions $\{a_\tau\}_{\tau=0}^{\infty}$ and the initial state is k_0, then the sequence of state variables is

$$h(k_t) = \omega_{20}\frac{1-\omega_{21}^t}{1-\omega_{21}} + \omega_{21}^t h(k_0) + \omega_{22}\sum_{\tau=0}^{t-1}\omega_{21}^{t-1-\tau}g(a_\tau).$$

The lifetime utility is given by

$$U(k_0, \{a_\tau\}_{\tau=0}^{\infty}) = \frac{1-\beta+\delta\beta}{1-\beta}\omega_{10} + \frac{\omega_{11}(1-\beta\omega_{21}+\delta\beta\omega_{21})}{1-\beta\omega_{21}}h(k_0) + V(\{a_\tau\}_{\tau=0}^{\infty}),$$

where

$$V(\{a_\tau\}_{\tau=0}^{\infty}) = \omega_{12}m(a_0) + \frac{\omega_{11}\omega_{22}\delta\beta}{1-\beta\omega_{21}}g(a_0) + \delta\sum_{j=1}^{\infty}\beta^j\left(\omega_{12}m(a_j) + \frac{\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}}g(a_j)\right).$$

Let $V_t \equiv V(\{a_\tau\}_{\tau=t}^{\infty})$, then

$$V_t = \omega_{12}m(a_t) + \frac{\delta\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}}g(a_t) + \beta V_{t+1} - \beta(1-\delta)\omega_{12}m(a_{t+1}).$$

In an organizational equilibrium, $V_t = V_{t+1} = \overline{V}$. In the steady state, $a_t = a_{t+1} = a^*$, and it follows

$$(1-\beta)\overline{V} = (1-\beta(1-\delta))\omega_{12}m(a^*) + \frac{\delta\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}}g(a^*).$$

The steady-state action a^* maximizes the steady-state action payoff $\overline{V}$, which solves

$$(1-\beta(1-\delta))\omega_{12}m_a(a^*) + \frac{\delta\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}}g_a(a^*) = 0.$$

By the requirement that $\overline{a} = a^*$ and the definition of $\boldsymbol{\omega}$, it follows that the stationary allocation has to satisfy

$$(1-\beta(1-\delta))u_a(\overline{k},\overline{a})(1-\beta F_k(\overline{k},\overline{a})) + \delta\beta u_k(\overline{k},\overline{a})F_a(\overline{k},\overline{a}) = 0.$$

Note that this condition characterizes the stationary allocation and is independent of the choice of $h(k), m(a), g(a)$.

Along the transition, the proposal function $a_{t+1} = q^*(a_t)$ is given by

$$a_{t+1} = m^{-1}\left\{\frac{\omega_{12}m(a_t) + \frac{\delta\beta\omega_{11}\omega_{22}}{1-\beta\omega_{21}}g(a_t) - (1-\beta)V^*}{\beta(1-\delta)\omega_{12}}\right\}$$

The proposal function in Section 4.2 is obtained by applying the corresponding function forms.

D Organizational Equilibrium in Policy Problems

D.1 A Description of the Game for Policy Applications

In Section 3, there is one player for each period. Here, the policymaker is still represented by one player for each period, but we also include a continuum of identical households that face a dynamic problem.[16]

The game unfolds as follows. In each period, the government in power takes an action $a \in A$ first. Then, the households move simultaneously. Each household takes an action $s \in S$. The aggregate state for next period evolves according to $k' = F(k, a, s)$. A full description would require us to specify what happens when households take different actions, so that, while they are identical ex ante, they may end up being different ex post. However, in most of the applications that are of interest, the household optimization problem has a unique solution. Hence, there can be no equilibrium in which identical households take different actions. Moreover, a deviation from a single household has no effect on aggregates. We exploit these properties and specify the evolution of the economy and preferences only after histories in which (almost) all households have taken the same action. Starting from an arbitrary period t and state k_t, household preferences are given by a function

$$Z(k_t, \{a_v, s_v, s_v^-\}_{v=t}^{\infty}), \tag{D.1}$$

where s_v represents the action taken by the individual household, and s_v^- is the action taken by (almost) all other households. We assume that S is a convex compact subset of a locally convex topological linear space and that Z is jointly continuous in all of its arguments (in the product topology), strictly quasiconcave in the own action sequence $\{s_v\}_{v=t}^{\infty}$, and weakly separable between the state and the remaining arguments. We also assume that household preferences are time consistent. More precisely, we assume that, given an initial level of the state k_t and a sequence of other households' actions $\{a_v, s_v\}_{v=t}^{\infty}$,

$$\begin{aligned} &Z(k_t, \{a_v, s_v, s_v\}_{v=t}^{\infty}) = \max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty}) \Longrightarrow Z(F(k_t, a_t, s_t), \{a_v, s_v, s_v\}_{v=t+1}^{\infty}) = \\ &\max_{\{\tilde{s}_v\}_{v=t+1}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t+1}^{\infty}). \end{aligned} \tag{D.2}$$

[16] The notion of an equilibrium can be readily extended to environments with finite types of households, or to economies with overlapping generations. Extending organizational equilibrium to environments can be done by interacting the analysis here with distributional notions of equilibrium as in Jovanovic and Rosenthal (1988).

Equation (D.2) states that, if it is optimal from period t to follow the same sequence of actions that all other households are taking, then it is also optimal to follow that sequence also in subsequent periods, as long as other households also continue to do the same. Notice that we exploit the fact that each household has no effect on the aggregates to leave the continuation preferences over several histories unspecified; this is convenient, because it prevents us from having to explicitly introduce individual state variables. To be concrete, consider the taxation game to which we apply this general definition; in that game, s_t is the individual saving rate. Equation (D.2) is written from the perspective of a household that starts with the same level of k_t as the aggregate, which allows us not to draw a distinction between the two. If that household finds it optimal to follow the same saving rate as all other households, then it will optimally choose to have the same level of k_{t+1}, and equation (D.2) ensures that the continuation plan will remain optimal from period $t+1$ onwards. If instead the household chooses a different saving rate from others, then it would potentially enter period $t+1$ with a different level of the state from the aggregate; however, whenever this choice does not maximize (D.1), we know this would not be an optimal individual choice without need to specify the entire continuation path; moreover, the individual deviation does not affect aggregate incentives, hence we do not need to keep track of it for the purpose of computing other households' best response either.

We define a competitive equilibrium from period t and a state k_t as a sequence $\{a_v, s_v\}_{v=t}^{\infty}$ such that

$$Z(k_t, \{a_v, s_v, s_v\}_{v=t}^{\infty}) = \max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty}).$$

Proposition 6. *Given any sequence of policy actions $\{a_v\}_{v=t}^{\infty}$, a competitive equilibrium exists.*

Proof. Fix k_t and $\{a_v\}_{v=t}^{\infty}$. Given our assumptions on S and Z, the best-response function

$$br(\{s_v\}_{v=0}^{\infty}) := \arg\max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty})$$

is well defined and continuous. By Brouwer's theorem, it admits a fixed point, which is a competitive equilibrium. □

Equation (D.2) ensures that the continuation of a competitive equilibrium is a competitive equilibrium itself. Also, the separability assumption about Z implies that, if $\{a_v, s_v\}_{v=t}^{\infty}$ is a competitive equilibrium from a state k_t, then it is also a competitive equilibrium from any other state k'_t.

In what follows, we proceed by assuming that the competitive equilibrium is unique given a sequence of policy actions, which can be verified in each specific application.[17]

At time t, government preferences are given by a function $\Psi^g(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots)$. We assume that this function is also weakly separable in k_t and its other arguments. For each given sequence of government actions $\{a_s\}_{s=t}^{\infty}$, a unique competitive equilibrium exists. The resulting sequence of private sector actions is given by a sequence $\{s_s\}_{s=t}^{\infty}$, which is independent of k_t, since household preferences are also separable in k_t. We thus specify the government utility from its sequence of actions as that experienced in the competitive equilibrium associated with those actions. With this specification, government preferences can be represented as in equation (3.1), and an organizational equilibrium can be defined in the same way as in Section 3. Existence of an organizational equilibrium is guaranteed by Proposition 2 when Assumptions 2 and 3 hold. However, these assumptions are significantly more restrictive in tax applications. As is well known, optimal tax problems frequently feature nonconvexities, in which case existence may have to be established in the specific application, as we do in our examples. Moreover, anticipation effects from the competitive equilibrium imply that Assumption 3 often does not hold either. It is worth noting that this assumption can be weakened. Its central role in our proof of Proposition 2 is to establish that the continuation sequence $(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)$ in equation (3.5) can be made independent of the current deviation a. In our tax example, we prove this result by showing instead that the static best-response $\arg\max_{\mathrm{a}} \mathrm{V}(\mathrm{a}, \mathrm{a}_0, \mathrm{a}_1, \mathrm{a}_2, \ldots)$ is independent of the sequence $\{a_t\}_{t=0}^{\infty}$: hence, any continuation which deters deviation to this action will also be sufficient to deter deviation to any other choice.

As we did for the simpler case of Section 3, we relate an organizational equilibrium to a strategic notion of equilibrium. To do so, we need to keep track of histories of play. A symmetric history of play is a record of all actions taken in the past; we distinguish between histories at which the government is called to play, which are given by $h^0 := \emptyset$ and

$$h^t := (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}), \quad t > 0,$$

and histories at which households are called to play, that take the form of $h^{p,0} := a_0$ and

$$h^{p,t} := (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}, a_t), \quad t > 0.$$

[17] Non-uniqueness can be accommodated by assuming a selection rule on how households coordinate when multiple equilibria are possible, as long as this rule has the properties that the continuation of a selected competitive equilibrium is selected itself as a continuation competitive equilibrium and that the selection is continuous.

Let H be the set of histories at which the government is called to play, and H^p the set of histories at which households are called to play. For the reasons discussed above, we only keep track of histories in which almost all households have taken the same action.

A strategy for the households is a mapping $\sigma^p : H^p \to S$; likewise, a government strategy is a mapping $\sigma : H \to A$. A *symmetric strategy profile* is a pair (σ^p, σ), representing how all households and the government will act following any symmetric history; it recursively induces a path of play $\{a_t, s_t\}_{t=0}^{\infty}$.

A symmetric strategy profile (σ^p, σ) is a sequential equilibrium if the following is true:

- Given that the government will follow σ and other households will follow σ^p, the actions dictated by σ^p are optimal for each household. After any history $h^{p,t}$, each household takes as given the government policy action a_t and the initial state k_t, which is recursively determined by the history of past play. Moreover, the strategy σ^p followed by other households and the government strategy σ determine the *future* path of aggregate play, $\{s_v, a_{v+1}\}_{v=t}^{\infty}$. Household optimality requires that the sequence of actions prescribed by σ^p is optimal along this path: equivalently stated, it requires the actions prescribed by σ^p to be a competitive equilibrium from period t on, following any arbitrary (symmetric) history.
- Given that households will follow the strategy σ^p and that future governments will follow the strategy σ, and given any past history h^t, the current government choice $\sigma(h^t)$ is optimal.

Proposition 7. *Given any organizational equilibrium, there exists a sequential equilibrium whose outcome coincides with the organizational equilibrium.*

Proof. Let $(a_0^*, a_1^*, a_2^*, ...)$ be an organizational equilibrium, and let $(s_0^*, s_1^*, ...)$ be the competitive-equilibrium associated with it. We construct a strategy profile recursively as follows:

- $\sigma(\emptyset) = a_0^*$;
- For any $t > 0$ and any history $h^t = (a_0, ..., a_{t-1})$ such that $a_s = a_s^* \quad \forall s = 0, ..., t-1$, $\sigma(h^t) = a_t^*$;
- For any $t > 0$ and any history $h^t = (a_0, ..., a_{t-1})$ such that $\exists s : a_s \neq a_s^*$, define $T := \max\{s < t : a_s \neq \sigma(a_0, ...a_{s-1})\}$ and set $\sigma(h^t) = a_{t-1-T}^*$.
- For any history $h_t^p = (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}, a_t)$ at which households are called to play, let $\{a_s^e\}_{s=t+1}^{\infty}$ be the sequence of government actions that follow from period $t+1$ if the government plays the continu-

ation of the strategy σ defined above following $(a_0, ..., a_t)$. Set $\sigma^p(h_t^p)$ to be the competitive equilibrium that is associated with $(a_t, a_{t+1}^e, a_{t+2}^e, ...)$, which exists and is unique by assumption.

By construction, the household strategy satisfies the second condition for a sequential equilibrium for any history of play. For the government, following any history, the strategy prescribes to play the organizational equilibrium sequence, either from its beginning or from some element a_t^*, $t > 0$. Should the government deviate from its strategy, the continuation strategy restarts the organizational equilibrium sequence from a_0^*. By the definition of an organizational equilibrium, continuing along the sequence is always weakly preferred to playing the best one-shot deviation followed by a restart; hence, the government optimality condition is satisfied and the strategy above describes a sequential equilibrium. □

Proposition 8. *Suppose that the government best response to a sequence of future unconditional policies by future governments is independent of the specific future sequence. Then the organizational equilibrium features a constant value over time, and there exists a symmetric sequential equilibrium in which the value for the government at each stage is independent of the past.*

Proof. The assumption in the proof is a weaker form of Assumption 3. It is sufficient to repeat the steps in the proof of Proposition 2. □

In Section 3, an organizational equilibrium effectively represents a refinement of an equilibrium based on specific beliefs that the single player at each stage entertains about future play. In the richer environment considered here, coordination of beliefs involves both the government and a continuum of private players. It is natural for this coordination to take the form of institutions and laws, which is why we call ours an “organizational equilibrium." Nonetheless, it is important to notice to contrast this role of institutions as purely coordinating expectations from an alternative, in which they represent forms of commitment. We take the view here that laws can be freely changed ex post and that government agencies can be reformed, so that they do not represent effective forms of commitment, and show how cooperation across different players over time can still be sustained, even when the self-interest of future players rules out the usual grim-trigger strategies.

E Proofs and Computational Details for Section 5

E.1 Proof of Lemma 1

First consider the following social planer's problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the resource constraint

$$c_t + k_{t+1} = k_t^{\alpha_t}.$$

Note that α_t in the production function can be time-varying in a deterministic fashion. The Euler condition is

$$\frac{1}{c_t} = \alpha_{t+1}\beta k_{t+1}^{\alpha_{t+1}-1}\frac{1}{c_{t+1}}.$$

Let μ_t denote the saving rate, i.e., $k_{t+1} = \mu_t k_t^{\alpha_t}$, then the Euler condition above can be rewritten as

$$\frac{1}{(1-\mu_t)z_t k_t^{\alpha_t}} = \alpha_{t+1}\beta k_{t+1}^{\alpha_{t+1}-1}\frac{1}{(1-\mu_{t+1})k_{t+1}^{\alpha_{t+1}}},$$

which can be further simplified to

$$\frac{\mu_t}{(1-\mu_t)} = \alpha_{t+1}\beta\frac{1}{(1-\mu_{t+1})}$$

The associated transversality condition is

$$\lim_{t\to\infty}\beta^t\frac{k_{t+1}}{c_t} = \lim_{t\to\infty}\beta^t\frac{\mu_t}{(1-\mu_t)} = 0.$$

By the standard concavity arguments, the planning problem has a unique solution and the Euler condition and the transaversality condition are necessary and sufficient for optimality. Hence, there must be a unique sequence of saving rates that satisfies them.

Now consider the tax-distorted competitive equilibrium in Section. In the tax-distorted competitive equilibrium, define φ_t as the after taxation saving rate, i.e., $k_{t+1} = \varphi_t(1-\alpha\tau_t)k_t^{\alpha}$, the Euler condition for households is

$$\frac{1}{c_t} = \alpha\beta k_{t+1}^{\alpha-1}\frac{(1-\tau_{t+1})}{c_{t+1}}$$

or

$$\frac{1}{(1-\varphi_t)(1-\alpha\tau_t)k_t^{\alpha}} = \alpha\beta k_{t+1}^{\alpha-1}\frac{(1-\tau_{t+1})}{(1-\varphi_{t+1})(1-\alpha\tau_{t+1})\varphi_t(1-\alpha\tau_t)k_t^{\alpha}}$$

which can be simplified to

$$\frac{\varphi_t}{(1-\varphi_t)} = \alpha\frac{1-\tau_{t+1}}{1-\alpha\tau_{t+1}}\beta\frac{1}{(1-\varphi_{t+1})}$$

The transversality condition is

$$\lim_{t\to\infty}\beta^t\frac{k_{t+1}}{c_t} = \lim_{t\to\infty}\beta^t\frac{\varphi_t}{(1-\varphi_t)} = 0$$

The Euler and the transversality condition must hold in the competitive equilibrium of the original economy. By defining $\alpha_t = \frac{1-\tau_{t+1}}{1-\alpha\tau_{t+1}}$, there exists a unique sequence of saving rates that satisfies them in the social planner's problem. As a result, there exists a unique competitive equilibrium.

E.2 Proof of Lemma 2

In an organizational equilibrium, the action payoff to government in different periods should be equalized. Therefore, for some constant $\overline{V}$,

$$\begin{aligned}
\overline{V} =& \sum_{j=0}^{\infty}\beta^j\left\{\log(1-\alpha\tau_{t+j}-s_{t+j})+\gamma\log\tau_{t+j}+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_{t+j}\right\}\\
=& \log(1-\alpha\tau_t-s_t)+\gamma\log\tau_t+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_t\\
&+\beta\sum_{j=0}^{\infty}\beta^j\left\{\log(1-\alpha\tau_{t+j+1}-s_{t+j+1})+\gamma\log\tau_{t+j+1}+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_{t+j+1}\right\}\\
=& \log(1-\alpha\tau_t-s_t)+\gamma\log\tau_t+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_t+\beta\overline{V}
\end{aligned}$$

It follows that for all t,

$$(1-\beta)\overline{V} = \log(1-\alpha\tau_t-s_t)+\gamma\log\tau_t+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_t$$

This leads to condition (5.6) and (5.7). In addition, the Euler condition for consumers needs to be satisfied, which leads to condition (5.8).

E.3 Proof of Proposition 5

Equation (5.10) simply rewrites condition (5.8) using the saving rate defined in equation (5.9). By Lemma (2), the sequence of tax rates derived from $q^*(\tau)$ together with the sequence of saving rates derived from $g(\tau; V^*)$ satisfy the Euler equation and that the action payoff to government in different periods is equalized. If $q^*(\tau)$ does not have a fixed point, then the tax rate will diverge to the upper or lower bound, which cannot be an equilibrium. If $q^*(\tau)$ has a fixed point, then V^* is equal to the highest payoff in the steady state. Therefore, V^* solves

$$V^* = \max_{\tau,s} \log(1 - \alpha\tau - s) + \gamma \log \tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s$$

subject to

$$s = (1-\tau)\alpha\beta$$

Any action payoff higher than V^* cannot yield a fixed point for $q^*(\tau)$.

Supposing the initial government wait for the next government to start the equilibrium $\{\tau_0, \tau_1, \ldots\}$ with associated saving rates $\{s_0, s_1, \ldots\}$, then the initial government's problem is

$$\max_{\tau} \quad \log(1 - \alpha\tau - s) + \gamma \log \tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s + \beta\overline{V}$$

subject to

$$\frac{s}{1-\alpha\tau - s} = \frac{\alpha\beta(1-\tau_0)}{1-\alpha\tau_0 - s_0}.$$

It follows that

$$s = \frac{\alpha\beta(1-\tau_0)}{1 - s_0 - \alpha\tau_0 + \alpha\beta(1-\tau_0)}(1-\alpha\tau)$$

and the relevant maximization problem becomes

$$\max_{\tau} \frac{1+\gamma}{1-\alpha\beta} \log(1-\alpha\tau) + \gamma \log \tau + \frac{1+\gamma}{1-\alpha\beta} \frac{\alpha\beta(1-\tau_0)}{1 - s_0 - \alpha\tau_0 + \alpha\beta(1-\tau_0)} + \beta\overline{V}$$

The optimal choice of the tax rate is

$$\tau = \frac{\gamma(1-\alpha\beta)}{\alpha(1+\gamma)}$$

The initial τ_0 can be determined by the no-delay condition in a straightforward way to make sure that the government does not benefit from sitting out.

E.4 Ramsey Outcome and Markov Equilibrium in the Taxation Problem

E.4.1 Simple Taxation Example

First consider the Markov equilibrium. We use a guess-and-verify approach. The guess is all future government will choose a constant tax rate τ' and future saving rate chosen by households is also a constant s'. The current government then choose her own tax rate taking τ' and s' as given

$$\max_{\tau} \log(1-\alpha\tau-s)+\gamma\log\tau+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s+\sum_{j=1}^{\infty}\beta^j\left\{\log(1-\alpha\tau'-s')+\gamma\log\tau'+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s'\right\},$$

subject to

$$\frac{s}{(1-s-\alpha\tau)}=\frac{\alpha\beta(1-\tau')}{1-s'-\alpha\tau'}.$$

Because τ' and s' are taken as given, the problem can be simplified to

$$\max_{\tau} \log(1-\alpha\tau-s)+\gamma\log\tau+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s.$$

By the Euler condition, we have

$$s=\frac{\alpha\beta(1-\tau')}{1-s'-\alpha\tau'+\alpha\beta(1-\tau')}(1-\alpha\tau)$$

It turns out that the current government always chooses a constant tax rate τ^M that is independent of τ',

$$\tau^M=\frac{\gamma(1-\alpha\beta)}{\alpha(1+\gamma)}$$

which verifies our guess.

Now turn to the Ramsey problem, which can be written as

$$\max_{\{s_t\},\{\tau_t\}}\sum_{t=0}^{\infty}\beta^t\log(1-\alpha\tau_t-s_t)+\gamma\log\tau_t+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_t$$

subject to

$$\frac{s_{t-1}}{1-s_{t-1}-\alpha\tau_{t-1}}=\frac{\alpha\beta(1-\tau_t)}{1-s_t-\alpha\tau_t},\quad \text{for } t>0$$

Note that

$$\log(1-\alpha\tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_t = \mathcal{H}(s_t, \tau_t).$$

Denote $\beta^t \lambda_t$ as the multiplier associated with the implementability constraint. The first-order conditions are

$$\begin{aligned}\mathcal{H}_s(t) &= \lambda_t \frac{\alpha\beta(1-\tau_t)}{(1-s_t-\alpha\tau_t)^2} - \beta\lambda_{t+1}\frac{1-\alpha\tau_t}{(1-s_t-\alpha\tau_t)^2}\\ \mathcal{H}_\tau(t) &= \lambda_t \frac{-\alpha\beta(1-s_t-\alpha)}{(1-s_t-\alpha\tau_t)^2} + \beta\lambda_{t+1}\frac{\alpha s_t}{(1-s_t-\alpha\tau_t)^2}\end{aligned}$$

In the steady state, $\lambda_t = \lambda_{t+1}$, and it follows that

$$\mathcal{H}_s = \frac{1}{\alpha}\mathcal{H}_\tau$$

E.4.2 Quantitative Taxation Model

In this appendix, we describe the case without capital depreciation deduction.[18] Let s_t denote the saving rate, i.e., $k_{t+1} = s_t f(k_t, \ell_t)$. The allocation in the competitive equilibrium is

$$\begin{aligned}k_{t+1} &= \overline{k} k_t^{1-\delta}(s_t y_t)^\delta,\\ g_t &= (\alpha(\tau_t^k + \tau_t) + (1-\alpha)(\tau_t^\ell + \tau_t))y_t,\\ c_t &= (1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^\ell + \tau_t))y_t.\end{aligned}$$

where $y_t = f(k_t, \ell_t)$ is the total output. The household's inter and intra Euler conditions satisfy

$$\begin{aligned}\frac{u_c(t)}{\delta\frac{k_{t+1}}{i_t}} =& \beta u_c(t+1)\left\{(1-\tau_{t+1}-\tau_{t+1}^k)f_k(k_{t+1},\ell_{t+1}) + \frac{1-\delta}{\delta}\frac{i_{t+1}}{k_{t+1}}\right\},\\ u_\ell(t) =& -u_c(t)(1-\tau_t^\ell-\tau_t)f_\ell(k_t,\ell_t),\end{aligned}$$

[18] When there is capital depreciation deduction, the households' budget constraint is

$$c_t + i_t = w_t\ell_t + r_t k_t - (\tau_t^\ell + \tau_t)w_t\ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k+\tau_t)}{r_t}\right)r_t k_t$$

However, this specification will break the weakly separable property. Instead, we assume that the budget constraint is

$$c_t + i_t = w_t\ell_t + r_t k_t - (\tau_t^\ell + \tau_t)w_t\ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k+\tau_t)}{\overline{r}}\right)r_t k_t$$

where $\overline{r}$ is the steady state interest rate. This specification will reserve the weakly separable property.

which can be written as

$$s_t = \delta\beta \frac{1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^\ell + \tau_t)}{1 - s_{t+1} - \alpha(\tau_{t+1}^k + \tau_{t+1}) - (1-\alpha)(\tau_{t+1}^\ell + \tau_t)} \left\{ \alpha(1 - \tau_{t+1}^k - \tau_{t+1}) + s_{t+1}\frac{1-\delta}{\delta} \right\},$$

$$\gamma_\ell \frac{\ell_t}{1-\ell_t} = \gamma_c \frac{(1-\alpha)(1-\tau_t^\ell - \tau_t)}{1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^\ell + \tau_t)}$$

Given an initial capital level $k_0 = k$, a sequence of saving rates, and a sequence of labor supply choices, the implied sequence of capital is

$$k_t = k_0^{(1-\delta+\alpha\delta)^t} \Pi_{j=0}^{t-1} s_j^{\delta(1-\delta+\alpha\delta)^{t-1-j}} \ell_j^{(1-\alpha)\delta(1-\delta+\alpha\delta)^{t-1-j}} \bar{k}^{1+(1-\delta+\alpha\delta)+\ldots+(1-\delta+\alpha\delta)^{t-1}}$$

Given a sequence of tax rates, the action payoff for the government is

$$V(\mathbf{s}, \boldsymbol{\tau}, \boldsymbol{\tau}^k, \boldsymbol{\tau}^\ell) = (\gamma_c + \gamma_g) \left\{ \frac{\alpha\beta\mu_2}{1 - (\mu_1 + \alpha\mu_2)\beta} \sum_{j=0}^{\infty} \beta^j \log s_j + \frac{(1-\alpha)(1-\beta\mu_1)}{1-(\mu_1+\alpha\mu_2)\beta} \sum_{j=0}^{\infty} \beta^j \log \ell_j \right\}$$
$$+ \sum_{j=0}^{\infty} \beta^j \gamma_c \log\left(1 - s_j - \alpha(\tau_j^k + \tau_j) - (1-\alpha)(\tau_j^\ell + \tau_j)\right) + \sum_{j=0}^{\infty} \beta^j \gamma_\ell \log(1 - \ell_j) + \sum_{j=0}^{\infty} \beta^j \gamma_g \log\left(\alpha(\tau_j^k + \tau_j) + (1-\alpha)(\tau_j^\ell + \tau_j)\right)$$

Ramsey Outcome Let $g_t = \mu_t f(k_t, \ell_t)$. The government budget constraint requires that

$$\mu_t = \alpha(\tau_t^k + \tau_t) + (1-\alpha)(\tau_t^\ell + \tau_t)$$

Depending on the tax instrument used for financing public spending, it is easy to define the required tax rate as a function of μ_t. Denote $\mathcal{T}^k(\mu), \mathcal{T}^\ell(\mu)$, and $\mathcal{T}(\mu)$ as the capital income, labor income, and total income tax rate to achieve the government spending to output ratio μ.

By the primal approach of the Ramsey problem, the government effectively chooses the sequence of saving rates, labor supply, and government spending to output ratios to maximize the welfare of the initial government

$$\max_{\{s_t\},\{\ell_t\},\{\mu_t\}} \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{\gamma_c + \gamma_g}{1 - (1-\delta+\alpha\delta)\beta} \left(\alpha\beta\delta \log s_t + (1-\alpha)(1-\beta(1-\delta)) \log \ell_t\right) + \gamma_c \log(1 - s_t - \mu_t)$$
$$+ \gamma_\ell \log(1-\ell_t) + \gamma_g \log(\mu_t) \Bigg)$$

subject to the corresponding implementability constraint

$$\frac{1}{\beta}\frac{s_t}{1-s_t-\mu_t} = \frac{\delta\alpha\left(1-(\mathcal{T}^k(\mu_{t+1})+\mathcal{T}(\mu_{t+1}))\chi\right)+(1-\delta)s_{t+1}}{1-s_{t+1}-\mu_{t+1}},$$
$$\gamma_\ell\frac{\ell_t}{1-\ell_t} = \gamma_c\frac{(1-\alpha)(1-\mathcal{T}^\ell(\mu_t)-\mathcal{T}(\mu_t))}{1-s_t-\mu_t}$$

Markov Equilibrium In the Markov equilibrium, the current government take future government's policy as given. In our setting, this will be that taking future government policy as a constant independent of current policy. Assume future tax rates are $\{\tau_f^k, \tau_f^\ell, \tau_f\}$ and the current policy choice is $\{\tau_0^k, \tau_0^\ell, \tau_0\}$. The current government action payoff is

$$\begin{aligned}
&M(\tau_0^k, \tau_0^\ell, \tau_0; \tau_f^k, \tau_f^\ell, \tau_f)\\
=&(\gamma_c+\gamma_g)\left\{\frac{\alpha\beta\mu_2}{1-(\mu_1+\alpha\mu_2)\beta}\left(\log s_0+\frac{\beta}{1-\beta}\log s_f\right)+\frac{(1-\alpha)(1-\beta\mu_1)}{1-(\mu_1+\alpha\mu_2)\beta}\left(\log\ell_0+\frac{\beta}{1-\beta}\log\ell_f\right)\right\}\\
&+\gamma_c\left\{\log\left(1-s_0-\alpha(\tau_0^k+\tau_0)-(1-\alpha)(\tau_0^\ell+\tau_0)\right)+\frac{\beta}{1-\beta}\log\left(1-s_f-\alpha(\tau_f^k+\tau_f)-(1-\alpha)(\tau_f^\ell+\tau_f)\right)\right\}\\
&+\gamma_\ell\left\{\log(1-\ell_0)+\frac{\beta}{1-\beta}\log(1-\ell_f)\right\}\\
&+\gamma_g\left\{\log\left(\alpha(\tau_0^k+\tau_0)+(1-\alpha)(\tau_0^\ell+\tau_0)\right)+\frac{\beta}{1-\beta}\log\left(\alpha(\tau_f^k+\tau_f)+(1-\alpha)(\tau_f^\ell+\tau_f)\right)\right\}
\end{aligned}$$

The current government's problem is

$$\max_{\tau_0^k,\tau_0^\ell,\tau_0} M(\tau_0^k, \tau_0^\ell, \tau_0; \tau_f^k, \tau_f^\ell, \tau_f)$$

subject to the implementability constraints

$$s_0 = \delta\beta\frac{1-s_0-\alpha(\tau_0^k+\tau_0)-(1-\alpha)(\tau_0^\ell+\tau_0)}{1-s_f-\alpha(\tau_f^k+\tau_f)-(1-\alpha)(\tau_f^\ell+\tau_f)}\left\{\alpha(1-\tau_f^k-\tau_f)+s_f\frac{1-\delta}{\delta}\right\},$$
$$\gamma_\ell\frac{\ell_0}{1-\ell_0} = \gamma_c\frac{(1-\alpha)(1-\tau_0^\ell-\tau_0)}{1-s_0-\alpha(\tau_0^k+\tau_0)-(1-\alpha)(\tau_0^\ell+\tau_0)}$$
$$s_f = \frac{\delta\alpha\beta(1-\tau_f^k-\tau_f)}{1-\beta(1-\delta)}$$
$$\gamma_\ell\frac{\ell_f}{1-\ell_f} = \gamma_c\frac{(1-\alpha)(1-\tau_f^\ell-\tau_f)}{1-s_f-\alpha(\tau_f^k+\tau_f)-(1-\alpha)(\tau_f^\ell+\tau_f)}$$

The Markov equilibrium is then the fixed point where future taxes and the current taxes are the same.